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Asymptotics for the eigenvalues of Toeplitz matrices with a symbol having a power singularity

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Abstract

The present work is devoted to the construction of an asymptotic expansion for the eigenvalues of a Toeplitz matrix $T_n(a)$ as n goes to infinity, with a continuous and real-valued symbol a having a power singularity of degree γ with $1 < \gamma < 2$, at one point. The resulting matrix is dense and its entries decrease slowly to zero when moving away from the main diagonal, we apply the so called *simple-loop* (SL) method for constructing and justifying a uniform asymptotic expansion for all the eigenvalues. Note however, that the considered symbol does not fully satisfy the conditions imposed in previous works, but only in a small neighborhood of the singularity point. In the present work: (i) We construct and justify the asymptotic formulas of the SL method for the eigenvalues $\lambda_j(T_n(a))$ with $j \geqslant \varepsilon n$, where the eigenvalues are arranged in nondecreasing order and ε is a sufficiently small fixed number. (ii) We show, with the help of numerical calculations, that the obtained formulas give good approximations in the case $j < \varepsilon n$. (iii) We numerically show that the main term of the asymptotics for eigenvalues with $j < \varepsilon n$, formally obtained from the formulas of the SL method, coincides with the main term of the asymptotics constructed and justified in the classical works of Widom and Parter.

CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

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