



PROFESSOR VLADISLAV V. KRAVCHENKO: A MATHEMATICIAN AND A FRIEND

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Published online: 23 August 2023
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Professor Vladislav V. Kravchenko, a Senior Researcher in the Department of Mathematics, Center for Research and Advanced Studies of the National Polytechnic Institute (Centro de Investigación y de Estudios Avanzados del IPN, CINVESTAV, Campus Querétaro, Mexico) celebrated his 55th birthday on February 15, 2023.

His recent breakthrough research is related to methods of solution of direct and inverse spectral problems which has led to new representations for solutions of linear ordinary differential equations with variable coefficients, to new representations for associated transformation (transmutation) operators, and eventually to simple, direct and most effective computational algorithms for solving direct and inverse spectral and scattering problems on finite and infinite intervals for regular and singular Sturm–Liouville equations. The problems which have always been considered as computationally challenging, all at once became quite simple and treatable within the unified approach developed by V. Kravchenko.

The first work by V. Kravchenko in this direction was his paper of 2008 [1], in which with the aid of the pseudo-analytic (or generalized analytic) function theory he developed what is now called the SPPS (spectral parameter power series) representation for solutions of the Sturm–Liouville equation. In fact, that work showed a simple and elegant way for computing Taylor coefficients of the solution considered as an entire function with respect to the spectral parameter. In a subsequent work developed jointly with R. Michael Porter [2] it was shown that this new representation led to a numerical method for solving direct Sturm–Liouville problems and as a consequence to an approach for solving a series of real-world

This review article is to honor the work of Professor Vladislav V. Kravchenko, a Senior Researcher in the research mathematical center CINVESTAV (Center for Research and Advanced Study, Mexico), on the occasion of his 55th birthday. It contains some biographical notes about V. Kravchenko, as well as a bird’s-eye view on his research during the last decades.

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problems using this simple technique, as was shown in [3–8] by Kravchenko and his coauthors, some of them being his PhD students, and later in a plethora of publications by many researchers from all over the world as can be verified in scientific databases by looking for citations of [1, 2] and subsequent papers. As examples of papers implementing the SPPS method in different applied fields, the following papers can be mentioned [31, 32] (applications in fluid dynamics), [33–39] (applications in quantum mechanics), [40, 41] (in electrodynamics), [42–45] (in hydroacoustics), [46] (in heat transfer), [47] (in electronics). The so-called SPPS method is even explained in the Wikipedia article dedicated to Sturm–Liouville theory: https://en.wikipedia.org/wiki/Sturm%E2%80%93Liouville_theory

In [9] Kravchenko, together with his PhD student H. Campos and his young colleague S. Torba, showed the close relation between the SPPS representation and the transformation (or transmutation) operators, which have constituted an important and classical object in spectral theory since the middle of the last century. This led to a number of interesting papers by Kravchenko and coauthors regarding possible applications of this fact for efficient construction of the transformation operators. We would emphasize the papers [10–13], in which the final, elegant results were obtained offering very useful series representations for the integral kernels of the transformation operators and as a corollary, new and marvelous series representations for solutions of regular and singular Sturm–Liouville equations satisfying initial conditions at a finite point or at infinity (the so-called Jost solutions).

These results led to incredibly simple and efficient methods for solving both direct and inverse spectral and scattering problems on finite and infinite intervals that were developed in [12–19]. Some of these developments were reported in the monographs [20, 21]. At present no other existing approach (analytical, asymptotic, or purely numerical) can compete in practical solving direct and inverse spectral problems (especially, on infinite intervals). This is a revolutionary breakthrough in the field, which has and will have many important implications including such as the practical and efficient realization of the inverse scattering transform method for solving evolutionary nonlinear equations, development of the nonlinear Fourier transform, important for the nonlinear optical wave propagation, and in many other directions of modern mathematical physics [22–29].

In particular, we note that the use of Spectral Parameter Power Series was developed by a team led by Vladislav Kravchenko in connection to the inverse scattering transform method for practical realization of initial problem for the non-linear the Korteweg-de-Vries equation [30]. In this paper, it is not only proposed an efficient method for solving this well-known problem (which is the subject of extensive literature) but is also provided its rigorous justification. In connection with the foregoing, we, colleagues and friends of Vladislav, want to draw attention to the fact that, as a mathematician, he is not only in a position to develop effective methods for solving specific problems, but also sees the possibility of using these methods in other areas of mathematics and various applications.

As a concluding remark we would like to mention that here we described only some of the recent results of Vladislav Kravchenko which impressed us the most. Prof. Kravchenko is an internationally recognized specialist in such fields as complex and hypercomplex analysis and its applications in mathematical physics (electrodynamics, quantum mechanics, etc.), boundary and spectral problems for partial differential equations. He has authored five books, and more than a hundred papers published in top ranking journals.

In addition, we would like to note that the scientific interests of Vladislav Kravchenko are quite wide, and he has reached high level results in other areas of mathematics, for example, being one of the recognized specialists in the field of quaternionic analysis, where he introduced and studied elegant reformulations on some equations of mathematical physics to acquire results not always obtainable by other methods (a presentation can be found not only in the individual papers, but also in the monograph [V. V. Kravchenko "Applied quaternionic analysis" Heldermann-Verlag, Lemgo, Research and Exposition in Mathematics Series, v. 28, 2003]).

At the moment, he is extremely active in research, dynamically continues to develop the described above mathematical theory, actively works with co-authors and students around the world, and at the same time participates in international large-scale projects. For instance, in 2018–2022 he was the scientific coordinator of the recently created Mathematical Center of the Southern Federal University. His efforts aim at promoting mathematics and mathematical knowledge and at creating new opportunities and academic mobility for young scientists, and he is focused on long-term scientific ambitious projects.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest. The authors have no competing interests to declare relevant to the content of this article.

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