Scalar fields in p-adic QFT

Joint work with W. A. Zúñiga and J. A. Vallejo

María Luisa Mendoza

Department of Mathematics
CINVESTAV

May 14, 2019

Goals

- In "M. L. Mendoza-Martínez, J. A. Vallejo, W. A. Zúñiga-Galindo. **Acausal quantum theory for non-Archimedean scalar fields** Reviews in Mathematical Physics. Vol. 31, No. 4 (2019)". We established that:
- We compute explicitly the fundamental solutions for $p-$adic pseudodifferential operators of Klein-Gordon Type.
- We present the second quantization of the solutions of these Klein-Gordon equations.
- Present the construction of a family of quantum scalar fields over a $p-$adic spacetime which satisfy $p-$adic analogues of the Gårding–Wightman axioms.

Goals

- In "M. L. Mendoza-Martínez, J. A. Vallejo, W. A. Zúñiga-Galindo. **Acausal quantum theory for non-Archimedean scalar fields** Reviews in Mathematical Physics. Vol. 31, No. 4 (2019)". We established that:
- We compute explicitly the fundamental solutions for $p-$adic pseudodifferential operators of Klein-Gordon Type.
- We present the second quantization of the solutions of these Klein-Gordon equations.
- Present the construction of a family of quantum scalar fields over a $p-$adic spacetime which satisfy $p-$adic analogues of the Gårding–Wightman axioms.

Goals

- In "M. L. Mendoza-Martínez, J. A. Vallejo, W. A. Zúñiga-Galindo. **Acausal quantum theory for non-Archimedean scalar fields** Reviews in Mathematical Physics. Vol. 31, No. 4 (2019)". We established that:
- We compute explicitly the fundamental solutions for $p-$adic pseudodifferential operators of Klein-Gordon Type.
- We present the second quantization of the solutions of these Klein-Gordon equations.
- Present the construction of a family of quantum scalar fields over a $p-$adic spacetime which satisfy $p-$adic analogues of the Gårding–Wightman axioms.

Goals

- In "M. L. Mendoza-Martínez, J. A. Vallejo, W. A. Zúñiga-Galindo. **Acausal quantum theory for non-Archimedean scalar fields** Reviews in Mathematical Physics. Vol. 31, No. 4 (2019)". We established that:
- We compute explicitly the fundamental solutions for $p-$adic pseudodifferential operators of Klein-Gordon Type.
- We present the second quantization of the solutions of these Klein-Gordon equations.
- Present the construction of a family of quantum scalar fields over a $p-$adic spacetime which satisfy $p-$adic analogues of the Gårding–Wightman axioms.

Physical Motivations

Two sets $S_1, S_2 \subset \mathbb{R}^4$ are called spacelike separated if $x \in S_1$ and $y \in S_2$ implies that $|x - y|^2 < 0$. If $f, g \in \mathcal{S}(\mathbb{R}^4)$ are spacelike

separated then $[\Phi(f), \Phi(g)] = 0$.

In the 80's I. Volovich proposed that spacetime on Planck distances has a non-Archimedean structure.

The acausal spacetime

Choose a prime p. We set

$$\mathfrak{B}(x, y) := x_0 y_0 - s x_1 y_1 - p x_2 y_2 + s p x_3 y_3,$$

where $s \in \mathbb{Z}$ is a quadratic non-residue modulo p, i.e, the congruence $x^2 \equiv \ s \ mod \ p$ does not have solution. Then $\mathfrak{B}(x, y)$ is a symmetric non-degenerate $\mathbb{Q}_p$-bilinear form on $\mathbb{Q}_p^4 \times \mathbb{Q}_p^4$ and

The acausal spacetime

Choose a prime p. We set

$$\mathfrak{B}(x, y) := x_0 y_0 - s x_1 y_1 - p x_2 y_2 + sp x_3 y_3,$$

where $s \in \mathbb{Z}$ is a quadratic non-residue modulo p, i.e, the congruence $x^2 \equiv \ s \ mod \ p$ does not have solution. Then $\mathfrak{B}(x, y)$ is a symmetric non-degenerate $\mathbb{Q}_p$-bilinear form on $\mathbb{Q}_p^4 \times \mathbb{Q}_p^4$ and

$\mathfrak{q}(x)$ is a non-degenerate quadratic form on $\mathbb{Q}_p^4$.

$$\mathfrak{q}(x) = \mathfrak{B}(x, x) := x_0^2 - s x_1^2 - p x_2^2 + sp x_3^2.$$

In addition, $q(x)$ is the unique (up to linear equivalence) elliptic quadratic form in dimension four.

The acausal spacetime

Choose a prime p. We set

$$\mathfrak{B}(x,y) := x_0y_0 - sx_1y_1 - px_2y_2 + spx_3y_3,$$

where $s \in \mathbb{Z}$ is a quadratic non-residue modulo p, i.e, the congruence $x^2 \equiv \ s \ mod \ p$ does not have solution. Then $\mathfrak{B}(x,y)$ is a symmetric non-degenerate $\mathbb{Q}_p$-bilinear form on $\mathbb{Q}_p^4 \times \mathbb{Q}_p^4$ and

$\mathfrak{q}(x)$ is a non-degenerate quadratic form on $\mathbb{Q}_p^4$.

$$\mathfrak{q}(x) = \mathfrak{B}(x,x) := x_0^2 - sx_1^2 - px_2^2 + spx_3^2.$$

In addition, $q(x)$ is the unique (up to linear equivalence) elliptic quadratic form in dimension four. Minkowski Fourier transform

$$(\mathcal{F}g)(k) \ = \ \int_{\mathbb{Q}_p^4} \chi_p(\mathfrak{B}(x,k))g(x)d\mu(x), \qquad d\mu(x) = C(\mathfrak{q})d^nx.$$

The acausal spacetime

The orthogonal group $O(\mathfrak{q})$ is defined to be

$$\begin{aligned} O(\mathfrak{q}) &= \{\Lambda \in GL_4(\mathbb{Q}_p) : \mathfrak{B}(\Lambda x, \Lambda y) = \mathfrak{B}(x, y)\} \\ &= \{\Lambda \in GL_4(\mathbb{Q}_p) : \Lambda^T G \Lambda = G\}. \end{aligned}$$

$$\text{where } G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -s & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & sp \end{bmatrix} \quad \mathfrak{q}(x) = x^T G x.$$

The acausal spacetime

The orthogonal group $O(\mathfrak{q})$ is defined to be

$$\begin{aligned} O(\mathfrak{q}) &= \{\Lambda \in GL_4(\mathbb{Q}_p) : \mathfrak{B}(\Lambda x, \Lambda y) = \mathfrak{B}(x, y)\} \\ &= \{\Lambda \in GL_4(\mathbb{Q}_p) : \Lambda^T G \Lambda = G\}. \end{aligned}$$

$$\text{where } G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -s & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & sp \end{bmatrix} \mathfrak{q}(x) = x^T G x.$$

For $t \in \mathbb{Q}_p^\times$, put $V_t := V_t(\mathfrak{q}) = \{k \in \mathbb{Q}_p^4 : \mathfrak{q}(k) = t\}$.

Then orthogonal group $O(\mathfrak{q})$ acts transitively on V_t.

The acausal spacetime

Set

$$V := \{k = (k_0, \mathbf{k}) \in \mathbb{Q}_p \times \mathbb{Q}_p^3; \mathfrak{q}(k) = 1\}.$$

$1 = \mathfrak{q}(k) = k_0^2 - \mathfrak{q}_0(k)$, where $\mathfrak{q}_0(k) = sk_1^2 + pk_2^2 - spk_3^2$
We now define in $U_\mathfrak{q} \subset \mathbb{Q}_p^3$, two analytic functions as follows:

$$\begin{array}{ccc} U_\mathfrak{q} & \rightarrow & \mathbb{Q}_p \\ \\ \mathbf{k} & \rightarrow & \pm\sqrt{1 + sk_1^2 + pk_2^2 - spk_3^2} =: \pm\sqrt{\omega(\mathbf{k})}, \end{array}$$

The acausal spacetime

To define positive and negative, we need a multiplicative character that takes two values

$$\begin{aligned} \mathbb{Q}_p^\times &\to \{1,-1\} \\ x &\to \pi(x) \end{aligned}$$

$x > 0$ if $\pi(x) = 1$ $\qquad$ $x < 0$ if $\pi(x) = -1$

The acausal spacetime

We define positive and negative mass shells $V^{\pm}$:

$$V^{+} = \left\{ (k_0, \mathbf{k}) \in V ; k_0 > 0 \text{ y } k_0 = \sqrt{\omega\left(\mathbf{k}\right)} \right\} ,$$
$$V^{-} = \left\{ (k_0, \mathbf{k}) \in V ; k_0 < 0 \text{ y } k_0 = -\sqrt{\omega\left(\mathbf{k}\right)} \right\} .$$

The acausal spacetime

We define positive and negative mass shells $V^{\pm}$:

$$V^{+} = \left\{ (k_0, \mathbf{k}) \in V; k_0 > 0 \text{ y } k_0 = \sqrt{\omega\left(\mathbf{k}\right)} \right\} ,$$
$$V^{-} = \left\{ (k_0, \mathbf{k}) \in V; k_0 < 0 \text{ y } k_0 = -\sqrt{\omega\left(\mathbf{k}\right)} \right\} .$$

$$V = V^{+} \bigsqcup V^{-} \bigsqcup W .$$

The acausal spacetime

We define positive and negative mass shells $V^{\pm}$:

$$V^{+} = \left\{(k_0, \mathbf{k}) \in V; k_0 > 0 \text{ y } k_0 = \sqrt{\omega(\mathbf{k})}\right\},$$

$$V^{-} = \left\{(k_0, \mathbf{k}) \in V; k_0 < 0 \text{ y } k_0 = -\sqrt{\omega(\mathbf{k})}\right\}.$$

$$V = V^{+} \bigsqcup V^{-} \bigsqcup W.$$

$$W = \left\{(k_0, \mathbf{k}) \in \mathbb{Z}_p^4; \mathfrak{q}(0, \mathbf{k}) = 1\right\}$$

The restricted $p-$adic Poincaré group

Definición

The restricted p-adic Lorentz group is

$$\mathcal{L}_{+}^{\uparrow} = \left\{ \Lambda \in \mathbf{O}(\mathfrak{q}); \Lambda\left(V^{\pm}\right) = V^{\pm} \right\}.$$

The restricted $p-$adic Poincaré group is $\mathcal{P}_{+}^{\uparrow}$ *the set of pairs* (a, Λ), *where* $a \in \mathbb{Q}_p^4$ *and* $\Lambda \in \mathcal{L}_{+}^{\uparrow}$, *with the group operation*

$$(a, \Lambda_1)(b, \Lambda_2) = (a + \Lambda_1 b, \Lambda_1 \Lambda_2).$$

The restricted $p-$adic Poincaré group

Definición

The restricted p-adic Lorentz group is

$$\mathcal{L}_+^{\uparrow} = \left\{\Lambda \in \mathbf{O}(\mathfrak{q}); \Lambda\left(V^{\pm}\right) = V^{\pm}\right\}.$$

The restricted $p-$adic Poincaré group is $\mathcal{P}_+^{\uparrow}$ the set of pairs (a, Λ), where $a \in \mathbb{Q}_p^4$ and $\Lambda \in \mathcal{L}_+^{\uparrow}$, with the group operation

$$(a, \Lambda_1)(b, \Lambda_2) = (a + \Lambda_1 b, \Lambda_1\Lambda_2).$$

The group $\mathcal{P}_+^{\uparrow}$ acts on $\mathbb{Q}_p^4$ through $(a, \Lambda)\,x = \Lambda x + a$.

Klein-Gordon type pseudodifferential equations

Definición

For $\alpha > 0$, $m \in \mathbb{Q}_p^\times$, and $\mathfrak{q}$ as before, let us put:

$$\Box_{\mathfrak{q},\alpha,m} = \mathcal{F}^{-1} \circ |\mathfrak{q} - m^2|_p^\alpha \circ \mathcal{F}, \tag{1}$$

Operators of this type are called (1), p-adic pseudodifferential Klein-Gordon operators.

Definición

We say that $E_{\mathfrak{q},\alpha} \in \mathcal{D}'_{\mathbb{C}}$ is a fundamental solution for

$$\Box_{\mathfrak{q},\alpha} u = \varphi, \tag{2}$$

if $u = E_{\mathfrak{q},\alpha} * \varphi$ is a solution to (2) in $\mathcal{D}'_{\mathbb{C}}$, for any $\varphi \in \mathcal{D}_{\mathbb{C}}$.

Fundamental solutions

Theorem (1) M. L. Mendoza, J. A. Vallejo, W. A. Zúñiga

There exist fundamental solutions $E_{\mathfrak{q},\alpha}$ to $\square_{\mathfrak{q},\alpha}$ which are invariant under the action $\mathbf{O}(\mathfrak{q})$. Moreover, the distributions $E_{\mathfrak{q},\alpha}$ satisfy:

(i)

$$\mathcal{F}(E_{\mathfrak{q},\alpha}) = \mathcal{F}(E^0_{\mathfrak{q},\alpha}) + C\delta(\mathfrak{q}-1), \qquad (3)$$

where C is a non-zero complex constant and $\mathcal{F}(E^0_{\mathfrak{q},\alpha})$, $\delta(\mathfrak{q}-1)$ are distributions invariant under $\mathbf{O}(\mathfrak{q})$.

(ii)

$$1_V\mathcal{F}(E_{\mathfrak{q},\alpha}) = C\delta(\mathfrak{q}-1). \qquad (4)$$

In particular, the restriction of $\mathcal{F}(E_{\mathfrak{q},\alpha})$ to V is unique up to the multiplication by a non-zero complex constant.

Fundamental Solutions for $p-$adic pseudodifferential Operators of Klein-Gordon Type

$\mathcal{F}\left[E^{0}_{\mathfrak{q},\alpha}\right]$ is a linear combination of distributions of any of the types

$$\int_{\mathbb{Q}_p^4\setminus V}|\mathfrak{q}(x)-1|_p^{-\alpha}\theta(x)\,d^4x \quad\text{or}\quad p^{\alpha}\int_{\mathbb{Z}_p}|u_0|_p^{-\alpha}(\Theta_b(u_0)-\Theta_b(0))\,du_0.$$

Fundamental Solutions for $p-$adic pseudodifferential Operators of Klein-Gordon Type

Now consider the non-homogeneous $p-$adic Klein-Gordon equation:

$$\Box_{\mathfrak{q},\alpha} u\left(t,\mathbf{x}\right) = h\left(t,\mathbf{x}\right), \tag{5}$$

where $(t,\mathbf{x}) \in \mathbb{Q}_p \times \mathbb{Q}_p^3$ and $h\left(t,\mathbf{x}\right) \in \mathcal{D}_{\mathbb{C}}(\mathbb{Q}_p \times \mathbb{Q}_p^3)$.

Theorem (3) M. L. Mendoza, J. A. Vallejo, W. A. Zúñiga

(i) The equation

$$\square_{\mathfrak{q},\alpha} u(t,\mathbf{x}) = 0 \tag{6}$$

admits plane wave solutions: $\chi_p \{-\mathcal{B}((t,\mathbf{x}),(E^{\pm},\boldsymbol{\kappa}))\}$ is a weak, solution of (6). Where $(E^{\pm},\boldsymbol{\kappa}) \in V^{\pm}$ with $E^{\pm} = \pm\sqrt{\omega(\boldsymbol{\kappa})}$.

(ii) The distributions

$$\int_{U_{\mathfrak{q}}} \chi_p \left\{-\mathcal{B}\left((t,\mathbf{x}),\left(\sqrt{\omega(\mathbf{k})},\mathbf{k}\right)\right)\right\} \frac{d^3\mathbf{k}}{\left|\sqrt{\omega(\mathbf{k})}\right|_p} + \int_{U_{\mathfrak{q}}} \chi_p \left\{\mathcal{B}\left((t,\mathbf{x}),\left(-\sqrt{\omega(\mathbf{k})},\mathbf{k}\right)\right)\right\} \frac{d^3\mathbf{k}}{\left|\sqrt{\omega(\mathbf{k})}\right|_p}$$

are the unique weak solutions of (6) invariant under $\mathcal{L}_+^{\uparrow}$.

Theorem (3) M. L. Mendoza, J. A. Vallejo, W. A. Zúñiga

(iii) The distributions

$$u(t,\mathbf{x};A,B,C)=E_{\mathfrak{q}}^{0}(t,\mathbf{x})*h(t,\mathbf{x})+$$
$$C\int_{U_{\mathfrak{q}}}\Big\{\chi_p\Big(-\sqrt{\omega(\mathbf{k})}t+\mathfrak{B}_0(\mathbf{k},\mathbf{x})\Big)A(\mathbf{k})+$$
$$\chi_p\Big(\sqrt{\omega(\mathbf{k})}t+\mathfrak{B}_0(\mathbf{k},\mathbf{x})\Big)B(\mathbf{k})\Big\}\times\frac{d^3\mathbf{k}}{\left|\sqrt{\omega(\mathbf{k})}\right|_p},$$

where C is a non-zero complex number, and $A(\mathbf{k})$, $B(\mathbf{k})\in\mathcal{D}_{\mathbb{C}}(\mathbb{Q}_p^3)$, are weak solutions of (5).

Nuclear Hilbert spaces

The construction of a suitable analog of the Schwartz test functions is of the utmost importance.

Definición

[*W. A. Zúñiga, 2017*]. *For* $f, g \in \mathcal{D}_{\mathbb{K}}$, *with* $\mathbb{K} = \mathbb{R}, \mathbb{C}$, *put:*

$$\langle f, g \rangle_l := \int_{\mathbb{Q}_p^4} [\xi]_p^l \widehat{f}(\xi) \overline{\widehat{g}}(\xi) d^4(\xi),$$

for $l \in \mathbb{N}$, *with the overbar denoting complex conjugate. Also,*

$$\mathcal{H}_\infty(\mathbb{Q}_p^4, \mathbb{K}) := \mathcal{H}_\infty(\mathbb{K}) = \bigcap_{l \in \mathbb{N}} \mathcal{H}_l(\mathbb{K}).$$

Nuclear Hilbert spaces

The mapping

$$\begin{array}{rccc} \Box_{\mathfrak{q},\alpha}: & \mathcal{H}_\infty(\mathbb{K}) & \rightarrow & \mathcal{H}_\infty(\mathbb{K}) \\ & h & \rightarrow & \Box_{\mathfrak{q},\alpha} h \end{array}$$

is a well-defined continuous linear operator between locally convex spaces.

Fock spaces

We define the symmetric Fock space over $\mathcal{H} = L^2_{\mathbb{C}}(V^+, d\lambda)$ as $\mathfrak{F}_s(\mathcal{H}) = \oplus_{n=0}^{\infty} \mathcal{H}_s^{(n)}$, where $\mathcal{H}_s^{(n)} = S_n \mathcal{H}^{(n)}$.

We denote by $S_n : \mathcal{H}^{(n)} \to S\mathcal{H}^{(n)}$, the symmetrization operator, and $S = \oplus_{n=0}^{\infty} S_n$

Fock spaces

We define the symmetric Fock space over $\mathcal{H} = L^2_{\mathbb{C}}(V^+, d\lambda)$ as $\mathfrak{F}_s(\mathcal{H}) = \oplus_{n=0}^{\infty} \mathcal{H}_s^{(n)}$, where $\mathcal{H}_s^{(n)} = S_n \mathcal{H}^{(n)}$.

We denote by $S_n : \mathcal{H}^{(n)} \to S\mathcal{H}^{(n)}$, the symmetrization operator, and $S = \oplus_{n=0}^{\infty} S_n \;\; \forall f \in \mathcal{H}_\infty(\mathbb{R})$. We define the operator $\mathbf{\Phi}$

$$\begin{array}{rcl} \mathbf{\Phi} : \mathcal{H}_\infty(\mathbb{R}) & \to & OP(\mathfrak{F}_s(\mathcal{H})) \\ f & \to & \mathbf{\Phi}_{\mathrm{S}}(Rf) \end{array}$$

Fock spaces

We define the symmetric Fock space over $\mathcal{H} = L^2_{\mathbb{C}}(V^+, d\lambda)$ as $\mathfrak{F}_s(\mathcal{H}) = \oplus_{n=0}^{\infty}\mathcal{H}_s^{(n)}$, where $\mathcal{H}_s^{(n)} = S_n\mathcal{H}^{(n)}$.

We denote by $S_n : \mathcal{H}^{(n)} \to S\mathcal{H}^{(n)}$, the symmetrization operator, and $S = \oplus_{n=0}^{\infty} S_n \;\; \forall f \in \mathcal{H}_\infty(\mathbb{R})$. We define the operator $\mathbf{\Phi}$

$$\begin{array}{rcl} \mathbf{\Phi} : \mathcal{H}_\infty(\mathbb{R}) & \to & OP(\mathfrak{F}_s(\mathcal{H})) \\ f & \to & \mathbf{\Phi}_{\mathrm{S}}(Rf) \end{array}$$

Let $H = \mathfrak{F}_s(L^2_{\mathbb{C}}(V^+, d\lambda))$, $\mathfrak{U} = \Gamma(U(\cdot,\cdot)) = \otimes_{k=1}^{n} U(\cdot,\cdot)$, where

$(U(a,\Lambda)\psi)(k) = \chi_p(\mathfrak{B}(a,k))\psi(\Lambda^{-1}k)$, $\mathbf{\Phi}$ and $D = F_0$.

A $p-$adic scalar QFT is a set $\{H, \mathfrak{U}, \mathbf{\Phi}, D\}$ satisfying

p-adic Gårding–Wightman axioms

1. Relativistic invariance of states: H is a separable Hilbert space and
$$\mathfrak{U}(\cdot,\cdot) : \mathcal{P}_{+}^{\uparrow} \longrightarrow U(H).$$
is a strongly continuous unitary representation.
2. Spectral condition: There exists a measure $E_{V^{+}}$ on $\mathbb{Q}_p^4$ corresponding to $\mathfrak{U}(a, I)$ supported on $\overline{S(V^{+})}$. (The topological closure of the additive semigroup generated by the vectors of V^{+}).
3. Existence of a vacuum. $\exists! \ \Upsilon_0 \in H \ni U(a, I)\,\Upsilon_0 = \Upsilon_0 \ \forall$ $a \in \mathbb{Q}_p^4$, this vector is called the *vacuum*.

p-adic Gårding–Wightman axioms

4. Invariant domains for the fields: $\exists\ D \subset H$ and a map from $\mathcal{H}_{\infty}(\mathbb{C})$ to the unbounded operators on H such that:
   (i) $\forall\ f \in \mathcal{H}_{\infty}(\mathbb{C})$, it is $D \subset Dom(\mathbf{\Phi}(f))$, $D \subset Dom\left(\mathbf{\Phi}(f)^{*}\right)$, and $\mathbf{\Phi}(f)^{*} \upharpoonright D = \mathbf{\Phi}\left(\overline{f}\right) \upharpoonright D$.
   (ii) $\Upsilon_0 \in D$, and $\mathbf{\Phi}(f)\, D \subset D\ \forall\ f \in \mathcal{H}_{\infty}(\mathbb{C})$.
   (iii) For any fixed $\psi \in D$, the mapping $f \rightarrow \mathbf{\Phi}(f)\,\psi$ is linear in f.
5. Regularity of fields: $\forall\ \psi_1,\ \psi_2 \in D$, the mapping

$$f \rightarrow \langle \psi_1, \mathbf{\Phi}(f)\,\psi_2 \rangle_H$$

lies in $\mathcal{H}^{*}_{\infty}(\mathbb{C})$.

p-adic Gårding–Wightman axioms

6. Poincaré invariance of the field.: $\forall\ (a,\Lambda)\in\mathcal{P}_{+}^{\uparrow}$, $\mathfrak{U}(a,\Lambda)D\subset D$, and $\forall\ f\in\mathcal{H}_{\infty}(\mathbb{C})$, $\psi\in D$,

$$\mathfrak{U}(a,\Lambda)\,\mathbf{\Phi}(f)\,\mathfrak{U}(a,\Lambda)^{-1}\psi=\mathbf{\Phi}((a,\Lambda)f)\,\psi,$$

7. Local causality. If f, $g\in\mathcal{D}_{\mathbb{C}}\left(\mathbb{Z}_{p}^{4}\right)$, then

$$[\mathbf{\Phi}(f),\mathbf{\Phi}(g)]\,\Psi=(\mathbf{\Phi}(f)\mathbf{\Phi}(g)-\mathbf{\Phi}(g)\,\mathbf{\Phi}(f))\,\Psi=0,\forall\ \Psi\in D.$$

8. Ciclicity of vaccum: The set D_0 of finite superpositions of vectors $\mathbf{\Phi}(f_1)\cdots\mathbf{\Phi}(f_n)\,\Upsilon_0$ is dense in H.

p-adic Gårding–Wightman axioms

Theorem (2) M. L. Mendoza, J. A. Vallejo, W. A. Zúñiga

(i) The set

$$\left\{\mathfrak{F}_s(L^2_{\mathbb{C}}\left(V^+, d\lambda\right)), \Gamma\left(U\left(\cdot,\cdot\right)\right), \mathbf{\Phi}, F_0\right\}$$

satisfy the $p-$adic Gårding–Wightman axioms.

(ii) For any $f \in \mathcal{H}_\infty\left(\mathbb{C}\right)$,

$$\mathbf{\Phi}\left(\Box_{\mathfrak{q},\alpha} f\right) = 0.$$

- M. L. Mendoza-Martínez, J. A. Vallejo, W. A. Zúñiga-Galindo: Acausal quantum theory for non-Archimedean scalar fields. Reviews in Mathematical Physics. Vol. 31, No. 4 , (2019).
- Khrennikov Andrei, Kozyrev Sergei, Zúñiga-Galindo W. A.: Ultrametric Equations and its Applications. Encyclopedia of Mathematics and its Applications (168). Cambridge University Press (2018)
- Zúñiga-Galindo W. A.: Pseudodifferential equations over non-Archimedean spaces. Lectures Notes in Mathematics 2174. Springer (2016).

Thank you for your attention. ☺

mmendoza@math.cinvestav.mx