# Centre for Research and Advanced Study at IPN

# Department of Mathematics

## Master' Degree Program Admission Examination

**November, 2004**

## 1. Linear Algebra

1.1 Let M2(R) the vector space formed by all square matrices [2 x 2]; consider also the linear transformation $T : \mathrm{M}_2(\mathbf{R}) \longrightarrow \mathrm{M}_2(\mathbf{R})$ given by T(A) = AB where

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

Calculate four eigen values of T; and provide an example of one related eigen vector to each one of the possible eigen values.

1.2 Consider the following linear transformation $T : \mathbf{R}^4 \longrightarrow \mathbf{R},$

$$T(u_1, u_2, u_3, u_4) = det \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 4 & 8 \\ 2 & 3 & 5 & 7 \end{bmatrix}$$

Find an orthogonal basis for the kernel of T.

Let $\mathrm{M}_3(\mathbf{R})$ be the vector space formed by all square matrices [3 by 3]. Consider a matrix A in $\mathrm{M}_3(\mathbf{R})$such that AB is equals to any other matrix B in matrix B $\mathrm{M}_3(\mathbf{R})$. Demonstrate that A is identically equals to zero.

## 2. Calculus

2.1 Find the general solution of the following differential equation>:

$$\frac{dy}{dx} = exp(y/x)$$

of the following function:

$$f(t) = \int_0^t \ln(s^2 + t^2)ds$$

2.3 Considering the expansion of the exponential function

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!},$$

demonstrate that $exp(x + y) = \exp(x)\exp(y)$.

## 3. Optional Problems

3.1 Demonstrate that each open set in the straight line is the union or at least enumerable of intervals open and disjoint to even numbers.

3.2 Let p be a prime number and G a group of cardinality p3 whose center is not cyclical. Prove that G must be abelian.

3.3 Find the number of roots of the equation $z^4 + 5z + 1 = 0$ inside the disc $|z| \leq 1$.

3.4 Demonstrate that the group of positive real numbers with the multiplication is isomorphic to the group of all real numbers with the sum.

Prove too, however, that the group of the positive rational numbers with the multiplication is NOT isomorphic to the group of all the rational numbers witht the sum.