Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

June 25, 2010

Instructions: Solve all problems in sections 1, 2 and all the ones you can in section 3. All solutions must be properly justified. You have 2 hours to complete the exam.

1. Linear Algebra

1.1 Which values $t \in \mathbb{R}$ make the following matrix non-reversible?

$$\begin{bmatrix} \cos\, t & -\mathrm{sen}\, t \\ \mathrm{sen}\, t & \cos t \end{bmatrix}$$

1.2 Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x, y, z) = (x - y + 4z, 3x + 2y - z, 2x + y - z).$$

Find the vectors $(z, x, y) \in \mathbb{R}^3$ and the constants $\lambda \in \mathbb{R}$ such that

$$T(x, y, z) = (\lambda x, \lambda y, \lambda z).$$

1.3 Let $n \in \mathbb{N}$ be fixed and be X the space for all the real polynomials of grade at least n. Provide a base X and express wichi transformations of the following two are linear of X in X.

$$p(x) \mapsto \frac{dp(x)}{dx} + x, \quad p(x) \mapsto \int_0^x p(y)dy,$$

2. Calculus

2.1 For which values $x \in \mathbb{R}$make the following sum convergent and what is the limit when $n \to \infty$?

$$\sum_{k=1}^{n} kx^k$$

2.2 Prove that the function $f(x) = \mathrm{sen}(x)$ satisfies the following relation

$$f(x) = x + \int_0^x (y - x) f(y) dy.$$

2.3 Use Taylor's series to calculate

$$\lim_{x \to 0} \left(\frac{1}{\mathrm{sen}(x)} - \frac{1}{x} \right).$$

3. Optional Problems

3.1 Calculate the complex integral $\int_0^{2\pi} e^{it} \cos(e^{it}) dt.$

3.2 Is the set of matrices a field

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}, \; a, b \in \mathbb{R},$$

with usual operations of sum and multiplication?

3.3 Express if the set $\{\frac{1}{n}, n = 1, 2, \ldots\}$ has an accumulation point (limit point) in the set (0, 1)