Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

June 18, 2007

Instructions: Solve all the problems in sections 1, 2 and all the ones you can in section 3. All solutions must be properly justified. The exam will last for two hours.

1. Linear Algebra

1.1 Consider $A = \begin{pmatrix} 4 & -3 \\ 1 & 0 \end{pmatrix}$. Find a non-singular matrix P such that

$$P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

and prove that

$$A^n = \frac{3^n - 1}{2} A + \frac{3 - 3^n}{2} I_2.$$

1.2 Let A be a matrix n x n given by

$$A = \begin{pmatrix} a & 1 & \dots & 1 \\ 1 & a & \dots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \dots & a \end{pmatrix}.$$

Prove that $\det(A) = (a-1)^{n-1}(a+n-1)$.

1.3 Find the basis for the solution space of the following system of equations.

$$\begin{aligned} x_1 - 4x_2 + 3x_3 - x_4 = 0 \\ 2x_1 - 8x_2 + 6x_3 - 2x_4 = 0 \end{aligned}$$

2. Calculus

2.1 Determine the convergence or divergence of the following integrals. If the integral is convergent, evaluate the integral.

$$(a).\quad \int_0^\infty \frac{dx}{(1+x)^2} \qquad\qquad (b).\quad \int_0^\infty \frac{dx}{\sqrt{1+x}}$$

2.2 Determine if the series $\sum_{n=1}^{\infty} \frac{(3n)! + 4^{n+1}}{(3n+1)!}$ is convergent or not.

2.3 Find the critical points of the function $f(x,y) = x^2 + y^2 + x^2y + 4$ and determine if they correspond to maximums, minimums and saddle point.

3. Optional Problems

2.4 Let $M_n(\mathbb{R})$, be the algebra of matrices n x n, with the sum, multiplication and product by usual scalars. Demonstrate that in $M_n(\mathbb{R})$ do not exist non-trivial bilateral ideals.

2.5 Enumerate all groups of order 8. Which of them are abelian?

2.6 Let $f : (X, d_1) \to (Y, d_2)$ an arbitrary function between metric spaces. Prove that f is continuous if and only if $\forall\, K \subset X$ is compact and $f\,|_K$ is continuous.