Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

July 10, 1998

1. Linear Algebra

1.1 Let V the subspace of $\mathbb{R}^4$ that is made of all of the solutions of the following system of homogenous linear equations :

$$\begin{aligned} 2x - y + 2z + w &= 0 \\ x + y + z - w &= 0 \\ 2x + 4z - w &= 0 \end{aligned}$$

Determine the basis for V.

1.2 Consider the following matrix:

$$A = \begin{pmatrix} 2 & -1 & 2 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -8 & 1 & -8 \end{pmatrix}$$

Find a basis for the image of the linear transformation $\mathrm{T}\colon \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ defined by A.

1.3 Consider $T(x.y, z) = (3x + 2y + 4z, 2x + 2z, 4x + 2y + 3z).$

a) Find the matrix representation of T with respect to the canonical basis.

b) Determine the appropriate values of T in a basis of subspaces of corresponding appropriate vectors.

2. Calculus

2.1 Calculate the limit

$$\lim_{x \to 1} \frac{x^3 - 2x + 1}{2x^3 - 3x^2 + 5x - 4}$$

2.2 Express if the following series converge and why.

$$\sum_{i=1}^{\infty} \frac{1}{n} \quad y \qquad \sum_{i=1}^{\infty} \frac{1}{n^2}$$

2.3 Consider $g:(1,\infty) \longrightarrow \mathbb{R}$ given by

$$g(x) = \int_1^{x^3} cos(1+\sqrt{t})dt$$

Calculate $g'(x)$

3. Optional problems

3.1 Let V be the vector space of matrices of order n x n with real entries and with usual matrix operations. Consider:

$$W = \{A \in V | A^t = -A\}$$

where At is the transverse of A. Is W a vector subpace of V?

3.2 Consider $f_n : (0, \infty) \longrightarrow \mathbb{R}$ given by $f_n(x) = x^n$, with $n = \ldots, -2, -1, 0, 1, 2, \ldots$. What values of n is f_n uniformly continuous?

3.3 Let $(X, \overline{d})$ a metric space and defined for $x, y \in X$ $\overline{d}(x, y) := \min \{1, d(x, y)\}.$

Prove that $\overline{d}$ is a metric on X.

3.4 Let $\{V_i\}_{i=1}^{\infty}$ be a succession of open sets in $\mathbb{R}^n$. Is $\bigcap_{i=1}^{\infty} V_i$ an open set in $\mathbb{R}^n$.

3.5 Prove that the quotient group $\mathbb{C}^*/S^1$ is isomorphic to $\mathbb{R}^+$, where $\mathbb{C}^* = \{z \in \mathbb{C} : z \neq 0\}, \mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ and $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ are considered as multiplicative groups.