Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

July 3, 2008

1. Linear Algebra

1.1 Let M be a matrix of 3 x 3 real entries such that $M^3 = I_{3\times 3}$ and $M \neq I_{3\times 3}$ where $I_{3\times 3}$ is the identity matrix of 3 x 3.

i Provide an example of a matrix M that satisfies these conditions.

1.2 Let $M \in M_{n\times n}(\mathbb{C})$ be a Hermitian matrix that satisfies the following condition:

$$M^5 + M^3 + M = I_{n\times n}.$$

Prove that $M = I_{n\times n}$.

1.3 A matrix is Hermitian if it is auto-adjoint, that is, it is equals to its conjugated transverse.

1.4 Let M be a matrix n x n with real entries and M^t its transverse. Prove that M^tM and M^t have the same range.

2. Calculus

2.1 Let f be a continuous function at [0, 1]. Calculate the following limit

$$\lim_{n\to\infty} \int_0^1 x^n f(x)dx.$$

2.2 What real numbers for $a \in (1, \infty)$ satisfies that $x^a \leq a^x$ for each $x \in (1, \infty)$?

2.3 Prove that the equation $ae^x = 1+x+\frac{x^2}{2}$ where a is a positive constant, has accurately a real root.

3. Optional Problems

3.1 Calculate the following integral:

$$\int_0^{2\pi} e^{(e^{ix}-ix)}dx.$$

Suggestion: Use Cauchy's integral formula for derivatives.

3.2 Let R be a set of complex numbers of the form

$$a + 3bi \ \text{ con } a, b \in \mathbb{Z}.$$

prove that R is a sub ring of C and an integer domain, but not a unique factorization domain.

3.3 Prove that each group of order p^2, with p a prime number, is abelian.

3.4 Prove o provide a counter-example. Each connected set, locally connected by trajectories of $\mathbb{R}^n$ is connected by trajectories.