Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

February 21, 2000

1. Linear Algebra

1.1 Find the appropriate values for the following matrix:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

1.2 Find a basis for the subspace of $\mathbb{R}^3$ generated by the vectors (1, 2, 1), (1, 3, 0), (0, 1, 1) and (1, 1, 2)

1.3 Let A and B be real matrices n x m and m x n, respectively. Prove that if m < n, then det $(AB) \neq 0$.

2. Calculus

2.1 For each real number a, let be

$$f_a : [0, \infty) \longrightarrow \mathbb{R} \text{ definida por } f_a(x) = e^{ax}.$$

Determine that the values for a in the function f_a is uniformly continuous on $[0, \infty)$.

2.2 For the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = x^4 - 2x^2$ calculate the local extremes, inflexion points and intervals that is increasing, decreasing, concave or convex. Use this information to sketch out your graph.

2.3 Calculate the integral of the line:

$$\int_\alpha \frac{xdy - ydx}{x^2 + y^2}$$

where $\alpha : [0, 2\pi] \longrightarrow \mathbb{R}^2$ is a curve given by $\alpha(t) = (\cos\ (t) =, \sin\ (t))$.

3. Optional Problems

3.1 Prove that each group of order 4 is isomorphic to $\mathbb{Z}^4$ or to $\mathbb{Z}^2 \times \mathbb{Z}^2$.

3.2 Let $\triangle = \{z \in \mathbb{C} | |z| < 1\}$ be the unit disk. Prove that if a holomorphic function $f : \triangle \backslash \{0\} \longrightarrow \mathbb{C}$ is bounded when $z \longmapsto 0,$ then f extends to a holomorphic function on $\triangle.$

3.3 Given sets A and B of $\mathbb{R}^n$ it is defined by $d(A, B) = \inf \{d(x, y) | x \in A,$ and $\in B\}.$ Prove that if A is compact, B is closed and $A igcap B = \phi,$ then $d(A, B) > 0.$ Provide an example that proves that if this condition is not met when A is only closed.

3.4 Provide an example of a succession of functions in $L_2(\mathbb{R})$ that accurately converges at 0 but that does not converge at 0 with norm $L_2.$