Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January 22, 2001

1. Linear Algebra

1.1 Consider the matrix:

$$A = \begin{pmatrix} -1 & -2 & 6 \\ -1 & 0 & 3 \\ -1 & -1 & 4 \end{pmatrix}$$

a) Determine the characteristic polynomial of A.
b) Is A similar to a diagonal matrix?

1.2 Let $p(x) = a_n x^n + \ldots + a_0$ be a polynomial of grade $n \geq 1$ with $a_0 \neq 0$ and let A be a square matrix. Prove that if p(A) = 0, then A is reversible.

1.3 Let V be the subspace of $\mathbb{R}^4$ that is made up of al of the solutions of the following system of homogenous linear equations

$$\begin{aligned} 2x - y + 2z + w &= 0 \\ x + y + z - w &= 0 \\ 2x \quad\quad - 4z - w &= 0 \end{aligned}$$

Determine a basis for V.

2. Calculus

2.1 Let $\pi_1 : \mathbb{R}^2 \longrightarrow \mathbb{R}$ the projection $\pi_1(x, y) = x.$ Is π_1 a closed function?

2.2 Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ given by

$$g(x) = \int_x^{x^2} \cos(1 + t)dt$$

Calculate g'(x)

2.3 Determine the dimensions of the rectangular prism with square basis of the minimal surface among all that have a fixed volume V.

3. Optional Problems

3.1 Let P be a set of solutions in $\mathbb{R}^n$ of he system of linear inequalities:

$$a_{11}x_1 + \ldots + a_{1n}x_n \leq b_1$$
$$\vdots$$
$$a_{m1}x_1 + \ldots + a_{mn}x_n \leq b_m$$

where a_{ij} and b_i are real numbers for all i, j. Suppose that $P \neq \theta$. Is P a closed set? Is P convex?

3.2 Which of the following statements are right? Justify your answer.

a) Every infinite cyclical group is isomorphic to the group $(\mathbb{Z}+)$ of the integers with the sum.
b) Two finite abelian groups of the same order are isomorphic.

3.3 Prove that any of two convex open sets of real straight line are homomorphic. Is this statement true if you change the real straight line for $\mathbb{R}^n$ when $n \geq 2$?. Justify your answer.

3.4 Prove that if a holomorphic function

$$f : \mathbb{C} \longrightarrow \mathbb{C} \text{ satisface } |f(z)| \leq C|z|^n$$

For every $z \in \mathbb{C}$ where C is a positive constant and n is some positive integer, then f is a polynomial of grade <= n.