Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January, 2005

1. Linear Algebra

1.1 Let $\mathrm{M}_2(\mathrm{R})$ be the vector space formed by all square matrices [2 x 2]; also consider the linear transformation $T : \mathrm{M}_2(\mathrm{R}) \longrightarrow \mathrm{M}_2(\mathrm{R})$ given by $T(A) =$ transverse of A. Calculate a basis for $\mathrm{M}_2(\mathrm{R})$ such that the transformation of T is represented by a diagonal matrix. What are the possible values for the diagonal?

1.2 Demonstrate that the compound space for all polynomials (with sum and standard multiplication) is a vector space. Also, determine what the dimension of the vector space of polynomials is.

1.3 Find a unitary vector on $\mathbb{R}^3$ that is orthogonal to vectors [2, 4, 8] and [3, 9, 27]

2. Calculus

2.1 Calculate the derivative of the function $\int_0^x e^{(x+t)^8}dt,$ with respect to x.

2.2 Provide an example of a non-bound succession of real numbers $\{x_n\}$ with at least two accumulation points.

2.3 Demonstrate Euler's theorem on homogeneous functions. A function $F(x, y, z)$ is homogenous of grade p if for any parameter λ you have:

$$F(\lambda x, \lambda y, \lambda z) = \lambda^p F(x, y, z).$$

Demonstrate that if $F(x, y, z)$ is homogenous of grade p, then:

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z} = pF(x, y, z).$$

3. Optional Problems

3.1 Demonstrate that each periodic rational function $\frac{p(z)}{q(z)}$ is constant.
3.2 Calculate what the fundamental group of the projective planar is RP^2.
3.3 Determine if there are non-abelian groups with a prime number of elements.
3.4 Let X be a locally connected topological space. Consider an open U of X and a connected component C of U. Demonstrate that $U \cap FrC = \emptyset$.