Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January 31, 2008

1. Linear Algebra

1.1 Let V be a vector space and $T : V \to V$ a linear transformation such that $T^2 = I_V$, where I_V denotes an identity transformation of V on V. Consider the following sets:

$$H_1 = \{v \in V | T(v) = v\}, H_2 = \{v \in V | T(v) = -v\}.$$

Demonstrate that H_1 and H_2 are subspaces of V such that $V = H_1 \oplus H_2$.

1.2 Let $T(x, y, z) = (3x + 2y + 4z, 2x + 2z, 2x + 2y + 3z)$ be a linear transformation of $\mathbb{R}^3$ on $\mathbb{R}^3$.

(i) Find the matrix representation of T with respect of the canonical basis of $\mathbb{R}^3$.

(ii) Determine the appropriate values of T and a basis for the subspaces of appropriate vectors corresponding to the eigen values.

1.3 Let V be the vector space of all matrices of 3 x 3 and let A be the following diagonal matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Calculus

2.1 Consider the following function:

$$F(x) = \int_0^2 sen((x + t)^2)dt$$

Calculate $\frac{dF(x)}{dx}\Big|_{x=0}$, the derivative of $F(x)$ with respect to x on zero.

2.2 Which number is greater 3^π or π^3?

Note: You can not use a calculator and you need to provide proof

2.3 You have a circle and a square of areas A1 and A2, respectively. Determine the possible maximum of A1 + A2, subject to the condition that the sum of the perimeters is constant and equals to 10.

3. Optional problems

3.1 Provide an example of demonstrate that there are no examples for each of the following groups:

1) A non-abelian group
2) A finite, non-cyclical abelian group
3) An infinite group with subgroups of index five,
4) A group G with a subgroup H non-normal
5) A group G with a subgroup H of index two that is not normal

3.2 Demonstrate that for each integer $x \in \mathbb{Z}$ the number $x^3 - x$ is a multiple of 3. Is it true that $x^4 - x$ is a multiple of 4 for each $x \in \mathbb{Z}$?

3.3 Find the number of roots of z^4+5z+1 inside of the unitary disc.

3.4 Demonstrate that the following limit exists:

$$\lim_{N\to\infty} \sum_{k=1}^{N} \frac{1}{k} - \ln(N).$$