Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January 10, 2000

1. Linear Algebra

1.1 Calculate the reverse of the following matrix.

$$A = \begin{pmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

1.2 Consider the following matrix

$$A = \begin{pmatrix} 2 & -1 & 2 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -8 & 1 & -8 \end{pmatrix}$$

a) Find the basis for the null space of A.
b) Find a basis for the column space of A.
c) Determine the unit of A and the range of A are the dimension of the spaces in the previous parenthesis.

1.3 Let A be a real reversible matrix. Prove that there are real symmetrical matrices defined positive and Q orthogonal (i.e. $QQ^t = I$) such $A = PQ$.
(Suggestion: Use the properties of AA^t)

2. Calculus

2.1 Find the derivative of the function F defined in [0, 1] for each of the following definitions:

$$\text{(a)}\ \ F(x) = \int_0^{x^2} (1+t^3)^{-1} dt,$$

$$\text{(b)}\ \ F(x) = \int_{x^2}^{x} \sqrt{1+t^2 dt},$$

2.2 Determine the integer values of p that converge in the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

2.3 Calculate the integral of the line $F(x,y) = (x^2, xy)$ along the trajectory of the origin at the point (1, 1) for the parabola $y = x^2$.

3. Optional problems

3.1 Prove that the quotient group $\mathbb{C}^*/S^1$ is isomorphic to $\mathbb{R}^+$ where $\mathbb{C}^* = \{z \in \mathbb{C} : z \neq 0\}, \mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ and $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ considered as multiplicative. Furthermore, provide a geometric interpretation of classes of equivalency of the quotient group $\mathbb{C}^*/S^1$.

3.2 Prove that any finite group of prime order is cyclical.

3.3 Find, except isomorphism, all groups of order 6.

3.4 Prove that a space of Hilbert of infinite dimension, the closed unit-ball is not compact.

3.5 Let $f : U \longrightarrow \mathbb{C}$ a defined holomorphic function on an open U of C. Prove that the real and imaginary parts of F are harmonic functions.