Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 14, 2006

Instructions: Solve all of the problems in sections number 1 and 2 and optionally the ones in section 3. All solutions must be properly justified. The exam will last for 2 hours.

1. Linear Algebra

1.1 For $\lambda \in \mathbb{R}$, what values make it reversible for the matrix:

$$A_\lambda = \begin{pmatrix} 1 & \lambda & 0 & 0 \\ \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \end{pmatrix} \qquad ?$$

1.2 Say which of the following are linear transformations. If true, describe ker *T*.

a) $T : \mathbb{R}^n \to \mathbb{R}^n, \quad T(v) = v + v_0,$ for a fixed vector $v_0 \in \mathbb{R}^n$.

b) $T : \mathbb{C}^n \to \mathbb{C}^n, \quad T(z_1, \ldots, z_n) = (\bar{z_1}, \ldots, \bar{z_n}).$

c) $T : \mathbb{R}^3 \to \mathbb{R}, \quad T(v) = \langle v \times v_0, v_1 \rangle,$ for fixed vectors $v_0, v_1 \in \mathbb{R}^3$.

d) $T : \mathbb{R}^n \to \mathbb{R}, \quad T(v) = \langle v, v \rangle.$

e) $T : \mathbb{R}^n \to \mathbb{R}^n, \quad T(x) = \langle x, a \rangle a,$ for $a \in \mathbb{R}^n$ fixed.

1.3 Let An be an n x n matrix given by

$$a_{ij} = \begin{cases} 0 & \text{if} \quad |i-j| > 1 \\ 1 & \text{if} \quad |i-j| = 1 \\ 2\cos\theta & \text{if} \quad i = j \end{cases}$$

if $\Delta_n = \det A_n$, prove that $\Delta_{n+2} - 2\cos\theta\,\Delta_{n+1} + \Delta_n = 0.$

2. Calculus

2.1 Find a polynomial f of the lesser grade in such a way that

$$f(x_1) = a_1 \qquad f(x_2) = a_2 \qquad f'(x_1) = b_1, \qquad f'(x_2) = b_2$$

where $x_1 \neq x_2$ and a_1, a_2, b_1, b_2 are real numbers given.

2.2 Determine if the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$ are convergent or not.

2.3 Find the point in the first quadrant, on the parabola y = 4 – x2, in such a a way that the triangle determined by the tangent of the parabola in that point and the coordinate axis has a minimum area.

3. Optional problems

3.1 Let A_n be the matrix of problem 1.3. Prove that $0 < \theta < \pi$, $\det A_n = \frac{\operatorname{sen}(n+1)\theta}{\operatorname{sen}\theta}$.

3.2 If γ is a circle with center 0 and radius 2, positively oriented, prove that:

$$\int_{\gamma} \frac{e^z}{z-1}\, dz = 2\pi i e$$

3.3 Provide examples of two groups of order 24, that are non-abelian and non-isomorphic.

3.4 Write down and prove the Brower Fixed Point Theorem.