Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 28, 2000

1. Linear Álgebra

1.1 Let A be an n x n matrix with real entries.

a) Prove that A is reversible if and only if its columns form a basis for R^n.
b) Prove that if the entries for A are within set {0, 1} and they are in such way that there is exactly one in each row and column, then A is reversible.

1.2 Let V_1 + V_2 be subspaces in a real vector space V of finite dimension.
Prove that $\dim(V_1 + \dim V_2 - \dim(V_1 \cap V_2)$.

1.3 Find a set of values $\theta \in \mathbb{R}$ that make the following matrix diagonalizable:

$$A(theta) = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

2. Calculus

2.1 Let (fn) be a succession of definite continuous real functions on an interval [a, b] that converge into a continuous function:

$$f : [a, b] \longrightarrow \mathbb{R}.$$

Prove that if convergence is uniform on [a, b] then:

$$\lim_{n \longrightarrow \infty} \int_a^b fn(x)dx = \int_a^b f(x)dx$$

2.2 Find dimensions in a straight circular cylinder of minimal surface from all of those of fixed volume V.

2.3 Let D be the bounded region by axis y and parabola $x = -4y^2 + 3$. Calculate the integral:

$$\int_D x^3 y \, dx \, dy$$

3. Optional problems

3.1 Prove that if K and N are subgroups of group G, with normal N in G, then $K/(N \cap K) \cong NK/N.$

3.2 Prove, using Schwarz lemma, that all holomorphic bijection between two discs of complex planar is formed by $f(z) = \frac{az+b}{cz+d}$ for some constants a, b, c, d.

3.3 Let A be a connected set, open and closed in a metric space X. Prove that A is a connected component of X.

3.4 Provide an example of set $E \subset \mathbb{R}$ which is a compact non-enumerable set of size zero. Justify its affirmation.