Centro de Investigación y de Estudios Avanzados del IPN
Departament of Mathematics

Admissions Examination to the Masters' Program

23 January 2012

Instructions: Solve all problems from sections 1 and 2 and as many as you can from section 3. All solutions should be justified appropriately. The examination lasts for three hours.

1. Linear algebra

1.1 Let n be a natural number and $A = (a_{ij})$, where

$$a_{ij} = \binom{i+j}{i},$$

for $0 \leq i, j \leq n$. Prove that A has an inverse and that all the entries of A^{-1} are integers.

1.2 Let A be an $n \times n$ matrix and $x \in \mathbb{R}$, each with positive real entries. Prove that if $A^2 x = x$, then $Ax = x$.

1.3 Prove that a matrix A is diagonalizable if and only if there exists a basis composed of eigenvalues of A.

2. Calculus

2.1 Prove that the series

$$\sum_{k=1}^{n} \frac{3^k k!}{k^k}$$

does not converge.

2.2 Let h be a continuous function and g a differentiable function on $\mathbb{R}$. Calculate the derivative of the function

$$f(x) = \int_0^{g(x)} h(t)\, dt.$$

2.3 Let $f : \mathbb{R}^2 \to R$ be a function such that $|f(x, y)| \leq |(x, y)|^2$. Prove that f is differentiable at $(0, 0)$.

2. Optional problems

3.1 Let G be a finite group such thet $|G|$ is not a multiple of 3. Suppose that $(a, b)^3 = a^3 b^3 A$ for all $a, b \in G$. Prove that G is abelian.

3.2 Prove that in $\mathbb{R}^n$, a set is compact if and only if it is closed and bounded. ¿Is this result true in any metric space?

3.3 Give an example of a function which is continuous on the irrationals and discontinuous on the rationals. Justify.