Research and Advanced Studies of IPN
Department of Mathematics

Examination of admission to the MSc

January 17, 2011

Directions: Solve all the problems of Sections 1 and 2, and if you can in Section 3. All solutions must be properly justified. The exam lasts 3 hours.

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1. **Linear Algebra**
   1.1. Let *A* be an *n* × *n* matrix with entries in the set {0, 1} which has exactly two ones in each column and two ones in each row. Give necessary and sufficient conditions for the rank of *A* is *n*.
   1.2. Let *A, B, n* × *m* and *m* × *n* matrices respectively. If *AB* = I_n, and *BA* = I_m prove that *n=m*. Where I_k is the identity matrix of size *k* × *k*.
   1.3. Let *A* be an *n* × *n* matrix over a field *k*, whose eigenvalues are $\lambda_1, \lambda_2, \ldots, \lambda_m$. Be E_{λ_i} the space of λ_i, for *i = 1,. . . , m*. Prove that if $\sum_{i=1}^{m} \dim\left(E_{\lambda_i}\right) = n$ then there exists a matrix *P* such that $P\,AP^{-1}$ is a diagonal matrix.
2. **Calculus**
   2.1. Prove that the series $\sum_{k=1}^{n} \frac{1}{k}$ not converge.
   2.2. Let *h* be a continuous function, and g a differentiable function in $\mathbb{R}$. Calculate the derivative of the function $f(x) = \int_0^{g(x)} h(t)dt$
   2.3. $Let\ f: \mathbb{R}^2 \to \mathbb{R}$ defined by $f\ (x, y) =\ | xy |^{\frac{1}{2}}$. Prove that *f* is not differentiable at (0,0)
3. **Optional problems**
   3.1. If $f\ (x) =\ x^3 - x$, prove that $f\ (n)$ is a multiple of 3 for all $n\ \in\ \mathbb{Z}$.
   3.2. Let *A* be a compact, and $f: A \to \mathbb{R}$ a continuous function. Prove that *f* takes a maximum value and minimum value in *A*.
   3.3. Give an example of a continuous function in the irrational and discontinuous at the rationals. Justify.
   3.4. Calculate the integral : $\int_0^{2\pi} e^{it} \cos(e^{it})\, dt$