Bergman-Toeplitz operators on periodic planar domains

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We study spectra of Toeplitz operators T_a with periodic symbols in Bergman spaces $A^2(\Pi)$ on unbounded periodic planar domains Π, which are defined as the union of infinitely many copies of the translated, bounded periodic cell ϖ. We introduce Floquet-transform techniques and prove a version of the band-gap-spectrum formula, which is well-known in the framework of periodic elliptic spectral problems and which describes the essential spectrum of T_a in terms of the spectra of a family of Toepliz-type operators $T_{a,\eta}$ in the cell ϖ, where η is the so-called Floquet variable.

As an application, we consider periodic domains Π_h containing thin geometric structures related to a small geometric parameter $h > 0$. We show how to construct a Toeplitz operator $T_{\mathsf{a}} : A^2(\Pi_h) \to A^2(\Pi_h)$ such that the essential spectrum of T_{a} contains disjoint components which approximatively coincide with any given finite set of real numbers. Moreover, our method provides a systematic and illustrative way how to construct such examples by using Toeplitz operators on the unit disc $\mathbb{D}$ e.g. with radial symbols.

Using a Riemann mapping one can then find a Toeplitz operator $T_b : A^2(\mathbb{D}) \to A^2(\mathbb{D})$ with a bounded symbol b and with the same spectral properties as T_{a}.