Analytic Continuation of Bergman Spaces on the Ball and Commutative C^*-algebras

Gestur Ólafsson, Louisiana State University

It has been know since 1976 that Bergaman spaces on bounded domain, initially defined as holomorphic L^2-functions with respect to certain probability measure can be defined for much bigger set of parameters. We will discuss the case of the ball $\mathrm{B}^n \subset \mathbb{C}^n$ where the Bergman spaces are first defined for $\sigma > n$, but then the definition is extended to all $\sigma > 0$. It can be shown for $n \geq \sigma > 0$ there is no probability measure on the ball such that the elements in the Bergman space are square integrable with respect to that measure. The question is then how to define Toeplitz operators for this set of parameters.

After introducing the analytic continuation in two different ways we will discuss Toeplitz operators for the extended set of parameters. Finally we discuss how invariance under action of maximal abelian subgroups of $\mathrm{SU}(1, n)$, well known for $\sigma > 0$, lead also to abelian C^* algebras in the extended situation.