One-variable and Multi-variable Integral Calculus over the Levi-Civita Field and Applications

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Abstract

In this talk, we present a Lebesgue-like measure theory and integration on the Levi-Civita field. After reviewing the algebraic and order structures and basic elements of calculus on the field, we introduce a measure that proves to be a natural generalization of the Lebesgue measure on the field of the real numbers and have similar properties. Then we introduce a family of simple functions from which we obtain a larger family of measurable functions and derive a simple characterization of such functions. We study the properties of measurable functions, we show how to integrate them over measurable sets of the Levi-Civita field, and we show that the resulting integral satisfies similar properties to those of the Lebesgue integral of classical Real Analysis.

Then we generalize the one-dimensional measure and integration theory to two and three dimensions, showing that the resulting measures and double and triple integrals have similar properties to those from Real Analysis. Finally, we introduce so-called delta functions which are piecewise analytic and integrable on the whole space with integral equal to 1 and which reduce to the Dirac delta function when restricted to real points; and we present simple applications of the theory.