ACCOTA 2014 International Workshop on **Combinatorial and Computational** Aspects of Optimization, Topology and Algebra

> Ixtapa-Zihuatanejo, Guerrero, Mexico November 24-28, 2014

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ACCOTA 2014

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ACCOTA 2014

November 24 – 28, 2014 Ixtapa‐Zihuatanejo, Guerrero, Mexico

Program

Sunrise 06:30 am Sunset 17:30 pm

POSTERS

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ABSTRACTS

Small bi-regular graphs of even girth

Martha Gabriela Araujo-Pardo *Instituto de Matem´aticas, UNAM* garaujo@math.unam.mx

Joint work with Geoffrey Exoo and Robert Jajcay.

A graph of girth *g* that contains vertices of degrees *r* and *m* is called a bi-regular graph and denoted by $({r, m}, g)$ -graph. In analogy with the *Cage Problem*, we seek the smallest $({r, m}, g)$ graphs for given parameters $2 \le r < m$, $g \ge 3$, called $({r, m}, g)$ cages.

Recently, Jajcay and Exoo, constructed an infinite family of ({*r*, *m*}, *g*)-cages for *m* much larger than *r* and odd girth *g* whose orders match a well-known lower bound given by Downs, Gould, Mitchem and Saba in 1981. Also they proved that a generalization of this result to bi-regular cages of even girth is impossible, because if the girth is even the bi-regular cages never match this lower bound.

In 2003, Yang and Liang, given a lower bound of the order of $({r}, m, 6)$ -cages and they constructed families of graphs that match this lower bound. In 2008, Araujo-Pardo, Balbuena, García Vázquez, Marcote and Valenzuela showed lower bounds for any even girth, and constructed more families of graphs that match the lower bound for $({r, m}, 6)$ -cages.

In this work, we summarize and improve some of these lower bounds for the orders of bi-regular cages of even girth and present a generalization of the odd girth construction to even girth that provides us with a new general upper bound on the order of graphs with girths of the form $g = 2t$, *t* odd. This construction gives us infinitely many $({r, m}, 6)$ -cages with sufficiently large *m*. We also determine a ({3, 4}, 10)-cage of order 82.

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Total domination multisubdivision number of a graph

Diana Avella Alaminos *Facultad de Ciencias, UNAM* avella@matematicas.unam.mx

This is a joint work with M. Dettlaff, M. Lemańska and R. Zuazua.

We define the total domination multisubdivision number of a nonempty graph *G*, msd*γ^t* (*G*), as the minimum positive integer *k* such that there exists an edge which must be subdivided *k* times to increase the total domination number of *G*. We show that for any connected graph *G* of order at least two, $\text{msd}_{\gamma_t}(G) \leq 3$, and for trees the total domination multisubdivision number is equal to the known total domination subdivision number. We also determine the total domination multisubdivision number for some classes of graphs and characterize trees T with $\operatorname{msd}_{\gamma_t}(T)=1.$

On a conjecture on the order of cages with a given girth pair

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Joint work with Julián Salas.

A (*k*; *g*, *h*)-graph is a *k*-regular graph of girth pair (*g*, *h*) where *g* is the girth of the graph, *h* is the length of a smallest cycle of different parity than *g* and *g* < *h*. A $(k; g, h)$ -cage is a $(k; g, h)$ -graph with the least possible number of vertices denoted by $n(k; g, h)$. In this talk we prove that $n(k; g, h) \leq n(k, h)$ for all $(k; g, h)$ cages when *g* is odd, and for for *g* even and *h* sufficiently large provided that a bipartite (k, g) -cage exists. This conjecture was posed by Harary and Kóvacs in [2]. Also we include some comment about the last obtained upper bounds on the order of (*k*; *g*, *h*) cages for $g = 6, 8, 12$ [1].

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Entropy of graphs and walk regularity

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The notion of walk entropy $S(G, b)$ for a graph *G* at the inverse temperature *b* was proposed recently in joint work with E. Estrada (2013) as a way of characterizing graphs using statistical mechanics concepts. We prove conjectures due to Benzi: a graph *G* with *n* vertices is walk-regular if and only if its walk entropy reaches a maximum $S(G, b) = \ln n$ for some $b > 0$, and Estrada: if *G* is not walk regular, then $S(G, b) < ln n$ (resp. $< ln n - \epsilon$, for some $\epsilon > 0$ if moreover *G* is not regular) for every $b > 0$. We give

examples and applications.

Drawing the Horton set in an integer grid of small size

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In 1978 Erd˝o asked whether every sufficiently large set of *n* points in the plane in general position contains the the vertices of an empty convex *k*-gon. Shortly after this was shown to be true for $k = 3, 4, 5$; the remaining case of empty convex six-gons remained open until 2007. In 1983 Horton provided an example (now known as the Horton set) without a convex empty sevengon. Ever since Horton sets have been a widely used family of point sets in Combinatorial Geometry. In this talk we study the problem of finding sets of points with integer coordinates and with the same order type as the Horton set, while minimizing the absolute value of the largest coordinate.

Implosive graphs: On indecomposable parallelizations of graphs

Alejandro Flores Méndez *Departamento de Matem´aticas, Cinvestav-IPN* flores.mendez.alejandro@gmail.com

Joint work with Isidoro Gitler and Enrique Reyes.

A graph *G* is called implosive if every parallelization *G ^w* that is indecomposable satisfies that $w \in \{0, 1\}^n$. We show some local properties of implosive graphs and we prove that the clique-sum of implosive graphs is implosive. Furthermore, we show that if an odd subdivision of *G* is implosive, then *G* is implosive. The converse is not true and a counterexample is included. We show that perfect graphs and theta-ring graphs are implosive. Since these families are closed under induced subgraphs, we study when their obstructions are implosive. In particular, we prove that odd holes, odd antiholes, prisms, thetas and even wheels are implosive. We characterize the odd wheels and the pyramids

that are implosive obstructions. In Algebra, a graph is implosive if and only if its Symbolic Rees algebra is a square free monomial algebra over the original variables.

What can be computed without communications?

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This is a joint work with Heger Arfaoui.

The main objective of this talk is to provide illustrative examples of distributed network computing tasks for which it is possible to design tight lower bounds for quantum algorithms without having to manipulate concepts from quantum mechanics, at all. As a case study, we address the following class of 2-player tasks. Alice (resp., Bob) receives a boolean *x* (resp., *y*) as input, and must return a boolean a (resp., b) as output. A game between Alice and Bob is defined by a pair (f, g) of boolean functions. The objective of Alice and Bob playing game (f, g) is, for every pair (x, y) of inputs, to output values a and b, respectively, satisfying $f(a, b) = g(x, y)$, in absence of any communication between the two players, but in presence of shared resources. The ability of the two players to solve the game then depends on the type of resources they share. It is known that, for the so-called CHSH game, i.e., for the game a XOR $b = x$ AND y, the ability for the players to use entangled quantum bits (qubits) helps. We show that, apart from the CHSH game, quantum correlations do not help, in the sense that, for every game not equivalent to the CHSH game, there exists a classical protocol (using shared randomness) whose probability of success is at least as large as the one of any protocol using quantum resources. This result holds for both worst case and average case analysis. It is achieved by considering a model stronger than quantum correlations, the non-signaling model, which subsumes quantum mechanics, but is far easier to handle.

Generalized asynchronous computability theorem

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Joint work with Petr Kuznetsov, Télécom ParisTech, INFRES, and Ciprian Manolescu, UCLA=Math.

We consider the models of distributed computation defined as subsets of the runs of the iterated immediate snapshot model. Given a task *T* and a model *M*, we provide topological conditions for *T* to be solvable in *M*.

When applied to the wait-free model, our conditions result in the celebrated Asynchronous Computability Theorem (ACT) of Herlihy and Shavit.

To demonstrate the utility of our characterization, we consider a task that has been shown earlier to admit a very complex *t*resilient solution. In contrast, our generalized computability theorem confirms its *t*-resilient solvability in a straightforward manner.

Delta-wye reducibility of non-planar graphs

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Joint work with E. Reyes and G.A. Sandoval.

Deltawye reducibility of graphs is a well-known area of study. Among its main problems of interest is that of determining classes of graphs for which every member is deltawye reducible to a graph containing a single vertex. Epifanov showed that the class of planar graphs have this property. This was extended to the class of graphs having crossing number one by Archdeacon, Colbourn, Gitler, and Provan. Reducibility for non-planar graphs has been in general difficult to establish. In fact few results are know, for example, it has been proven for projective planar graphs,

graphs with no *K*⁵ minor and graphs with no *K*3,3 minor. The larger problem of determining the entire class of graphs that are deltawye reducible to a graph containing a single vertex remains open. For this last problem, Truemper showed that the class is minor closed and Yu has found a very large list of excluded minors for this class. Another important problem, because of its applications in other areas of graph theory is deltawye reducibility while maintaining a small set of "terminal" nodes; for research along these lines, see papers by Epifanov, Feo and Provan, Archdeacon et al, Gitler and Sagols, and Lino Demasi among other. In this talk we study delta-wye reducibility of families of other non-planar graphs, such as toroidal and almost-planar graphs, this last class recently shown to be reducible by Donald K. Wagner. In fact Wagner considers the following problem: for a given class *C* of graphs and a "target" set $T \subseteq C$, show that every member of *C* is deltawye reducible to a member of *T* with the added condition that every graph of some reduction sequence must belong to *C*. We give a different proof of this result and efficient algorithms for reducibility based in terminal algorithms for the planar case.

Union of kernel perfect digraphs

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A kernel of a directed graph, *D*, is an independent and absorbent set of vertices, *K*. A digraph is said to be kernel-perfect if and only if any induced subdigraph has a kernel. A classical result obtained by Sands, Sauer and Woodrow asserts that the union of two transitive digraphs is a kernel-perfect digraph. Later it was studied the union of a right-pretransitive and a left-pretransitive digraphs and how it becomes a kernel-perfect digraph. In this work we obtain sufficient conditions for the union of two kernelperfect digraphs to become a kernel-perfect digraph.

The Murty-Simon conjecture seen through the perspective of 3**-total-dominating critical graphs**

Adriana Hansberg *Instituto de Matem´aticas, UNAM* ahansberg@im.unam.mx

A graph is called 2-diameter critical if its diameter is 2 and the deletion of any of its edges results in a graph with diameter > 2 . Murty and Simon conjectured that the number of edges in a 2 diameter critical graph is at most *ⁿ* 2 $\frac{1}{4}$, being the balanced bipartite graphs the extremal graphs. When considering the complement of the graph, this problem can be restated in terms of "total domination". In this talk, I will present some resent results concerning this conjecture though the perspective of the so-called 3-total-dominating critical graphs.

Improving the Moore bound for the order of cages via counting cycles

Tatiana Jajcayova *Department of Applied Computer Science, Faculty of Mathematics, Physics and Computer Science, Comenius University* tatiana.jajcayova@fmph.uniba.sk

Despite being a very bad predictor of the order of cages, the wellknown Moore bound is still essentially the only general lower bound and the most it has been improved by is by two vertices. Based on the observation that in the Moore graphs the number of cycles of length close to the girth is independent of the vertex through which they pass, we devise a method for improving the Moore bound for certain classes of the girth/degree parameter pairs.

Improving the Moore bound for vertex-transitive graphs

Robert Jajcay *Department of Algebra, Faculty of Mathematics, Physics and Computer Science, Comenius University* robert.jajcay@fmph.uniba.sk

The well-known Moore bound serves both an upper bound for the order of *k*-regular graphs of given diameter and a lower bound for the order of *k*-regular graphs of given girth. Even though no constructions of graphs from either class come even close to this theoretical lower bound, no significant improvements are known. In our presentation, we report on improvements on the Moore bound for the case of vertex-transitive graphs; both for the degree/diameter and the degree/girth versions of the problem. We view our results as further evidence toward showing that the Moore bound is rarely a good predictor of the orders of the extremal graphs.

From linear codes to automorphisms of Latin squares: An unusual extremal problem for set systems

Gyula Károlyi *R´enyi Institute and E¨otv¨os University* karolyi@cs.elte.hu

It is known that the number of automorphism of a nontrivial group is always smaller than the order of the group. For Latin squares (that is, for quasigroups) this is not true. The first quasigroup with large automorphism group was constructed recently by McKay, Wanless and Zhang. They also proved a quadratic upper bound.

A key element in the construction is finding a set system $\mathcal F$ with the following properties: For every pair $A \neq B$ of sets in F there exist at least $t \geq 2$ different sets $C \in \mathcal{F}$ such that $A \cup B = A \cup C =$ *B* ∪ *C*. Moreover, the size of the largest set in $\mathcal F$ must be small compared to the size of the union of the elements in $\mathcal F$. Denoting the latter by *n*, let the function $f_t(n)$ describe the smallest possible

size of the largest element in \mathcal{F} .

It is easy to see that $n/2 < f(n) \leq f_t(n)$. Moreover, $f_t(n) < cn$ for any fixed constant *c* > 2/3 for *n* large enough. We determine the exact value of $f(n)$ in terms of the so-called meta-Fibonacci sequence and prove that

$$
f_t(n) = n/2 + O_t(\sqrt{n \log n}),
$$

which gives hope to get close to the above mentioned quadratic bound.

On achromatic and pseudoachromatic indices of finite spaces

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Joint work with G. Araujo-Pardo, C. Rubio-Montiel and A. Vázquez-Avila. ´

A *decomposition* of a simple graph $G = (V(G), E(G))$ is a pair [*G*, D] where D is a set of induced subgraphs of *G*, such that every edge of *G* belongs to exactly one subgraph in D. A *coloring* of a decomposition $[G, \mathcal{D}]$ with *k* colors is a surjective function that assigns to edges of *G* a color from a *k*-set of colors, such that all edges of $H \in \mathcal{D}$ have the same color. A coloring of $[G, \mathcal{D}]$ with *k* colors is *proper*, if for all $H_1, H_2 \in \mathcal{D}$ with $H_1 \neq H_2$ and $V(H_1) \cap V(H_2) \neq \emptyset$, then $E(H_1)$ and $E(H_2)$ have different colors. The *chromatic index* $\chi'([G,\mathcal{D})]$ of a decomposition is the smallest number *k* for which there exists a proper coloring of $[G, \mathcal{D}]$ with *k* colors. A coloring of $[G, \mathcal{D}]$ with *k* colors is *complete* if each pair of colors appears on at least a vertex of *G*. The *pseudoachromatic index* $\psi'([G,\mathcal{D}])$ of a decomposition is the largest number *k* for which there exist a complete coloring with *k* colors. The *achromatic index α* ′ ([*G*, D]) of a decomposition is the largest number *k* for which there exist a proper and complete coloring with *k* colors. If $\mathcal{D} = E(G)$ then $\chi'([G,E])$, $\alpha'([G,E])$ and *ψ* ′ ([*G*, *E*]) are the usual *chromatic*, *achromatic* and *pseudoachromatic indices* of *G*, respectively.

Clearly we have that

 $\chi'([G,\mathcal{D}]) \leq \alpha'([G,\mathcal{D}]) \leq \psi'([G,\mathcal{D}]).$

Designs define decompositions of the corresponding complete graphs in the natural way. Identify the points of a (v, κ) -design *D* with the set of vertices of the complete graph *Kv*. Then the set of points of each block of *D* induces in *K^v* a subgraph isomorphic to *K^κ* and these subgraphs give a decomposition of *Kv*.

In this talk we consider some decompositions of the complete graphs coming from the line-sets of finite affine and projective spaces $AG(n, q)$ and $PG(n, q)$. We determine the exact values of the achromatic and pseudoachromatic indices in the planar cases and give estimates on the indices if $n > 2$.

Diagonal flips in 4-connected triangulations on surfaces

Naoki Matsumoto *Research fellow of the Japan society for the promotion of science, Yokohama National University* naoki.matsumo10@gmail.com

A *triangulation* on a closed surface *F* 2 is a simple graph embedded on *F* ² with each face triangular. In this talk, we consider a local transformation in 4-connected triangulations on closed surfaces, called a diagonal flip.

Eigenvalues of tournaments

Bojan Mohar *Simon Fraser University & IMFM* mohar@sfu.ca

There are different ways of defining eigenvalues of digraphs. We will compare these possibilities for the case of tournaments.

Heterochromatic and gracefull spanning trees

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Joint work with Eduardo Rivera-Campo and Ricardo Strausz.

We present a characterization of the edge-colorings of graphs which produce an spanning tree with no pair of its edges of the same color (an heterochromatic spanning tree). Also a relation of this kind of edge-colorigns with the problem of the gracefull trees is presented.

A formula for eulerian and Simon Newcomb's numbers

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Let recall the Simon Newcomb's problem: Consider a deck S of *N* cards containing b_i cards of face value *i*, for $i = 1, ..., n$ so that $b_1 + ... + b_n = N$. Turn over the top card and put it down face up. Turn over the second card and place it on top of the first card if the face value of the second card is less than or equal to the first - otherwise start a new pile. Continue this process until all *N* cards have been turned over. The number of possible cases with *k* piles or rises so formed yields the Simon Newcomb number $A([\mathbf{b}], k)$, with specification $[\mathbf{b}] = [b_1, ..., b_n]$. In this talk we solve the original question of Simon Newcomb by given a formula for the Simon Newcomb's numbers *A*([**b**], *k*) involving only positive integer numbers. The main point consist to translate this problem into a problem of *h*−vectors, and use the work of Stanley and Björner.

Algebraic topology for digraphs

Yury Muranov *University of Warmia and Mazury* ymuranov@mail.ru

In the talk I would like to present recent results on algebraic topology for the category of digraphs obtained jointly with Alexander Grigoryan (University of Bielefeld) and Shing-Tung Yau (Harvard University). The homotopy theory of undirected graphs was constructed by Barcelo, Kramer, Laubenbacher, Weaver, and Longueville. We define a homotopy and homology theories of digraphs and prove the invariance of the homology theory under homotopy and prove that the relation between fundamental group and the first homology group is similar to this one in the classical algebraic topology. The category of graphs can be identified by a natural way with a full subcategory of digraphs. Thus we obtain also a homology and homotopy theory for graphs. Our homology theory of graphs is new and its construction realizes the desire of Babson, Barcelo, Longueville, and Laubenbacher "for a homology theory associated to the A-theory of a graph". We describe relations between described homotopy theory of graphs and the corresponding results of predecessors. We present results about behavior of homology groups for several natural constructions in the category of digraphs and provide a number of examples.

Book embedding of graphs embeddable on surfaces

Atsuhiro Nakamoto *Yokohama National University* nakamoto@ynu.ac.jp

It is known that every planar graph is 4-page embeddable, and that every toroidal graph is 7-page embeddable. In our talk, we give an upper bound for the page number of projective planar graphs, and that for graphs which have a locally planar embedding on surfaces.

Chromatic numbers and algebraic structure of graphs on surfaces

Kenta Noguchi *Keio University* knoguchi@comb.math.keio.ac.jp

In the graph theory, Four Color Theorem is one of the most famous problem, that is, a problem determining the number of colors to color any maps so that adjacent countries receive different colors. This is solved affirmatively [1] and called "Four Color Theorem." It is equivalent to the following statement; every planar graph is 4-colorable. We now consider the coloring problem of graphs on general surfaces.

It is known that the upper bound of chromatic numbers of graphs on a surface F^2 becomes larger depending on the genus of F^2 . But it is also known that there is a family of graphs on surfaces, called locally planar graphs, which has the chromatic number at most 5. In this talk, we introduce a classification of locally planar graphs by using algebraic structure, called a cycle parity and a monodromy.

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Generalized tournaments and CKI-digraphs

Mika Olsen *UAM-Cuajimalpa* olsen@correo.cua.uam.mx

Generalizations of tournaments have obtained a lot of attention during the last 25 years, an interesting survey of generalizations of tournaments can be found in [1]. Some classes of generalizations of tournaments preserve nice structures and important properties of tournaments, for instance, the digraphs of some classes of generalizations of tournaments are hamiltonian digraphs (or has a hamiltonian path when it is not strongly connected) or the digraphs are pancyclic or satisfies the path partition conjecture. Some classes of generalizations of tournaments have been characterized en terms of its underlying graph or in terms of the structure and for other classes no nice characterizations are known yet. The quasi-transitive digraphs are digraphs such that whenever there is a path *uvw*, then $\{uw, wu\} \cup A(D) \neq \emptyset$ and the semicomplete multipartite tournaments are those, which underlying graph is a complete multipartite graph. The locally semicomplete digraphs where introduced by Bang-Jensen as generalization of tournaments, and a digraph *D* is locally semicomplete if for every vertex of *D* the in-neighborhood and the outneighborhood induces a semicomplete digraph. The round digraphs is a proper subclass of the locally semicomplete digraphs. The quasi-transitive digraphs, the semicomplete multipartite tournaments, the round digraphs and the locally semicomplete digraphs are classes of generalizations of tournaments which have been characterized. The families of H_i -free digraphs, $i = 1, 2, 3, 4$, where introduced by Bang-Jensen in 2004, as a generalization of semicomplete bipartite digraphs. A not necessarily directed path *P* = *uvwx* is an *H*₁-path if $u \to v \leftarrow w \leftarrow x$; an *H*₂-path if $u \leftarrow v \leftarrow w \rightarrow x$; an *H*₃-path if *P* is a directed path and an *H*₄path if *P* is an antidirected path (*P* has no directed subpath on two vertices). For $i = 1, 2, 3, 4$, Bang-Jensen defined a digraph to be *Hⁱ* -free, if every *Hⁱ* -path *uvwx* has an arc between *u* and *x*. The family of H_1 -free digraphs (H_2 -free digraphs, resp.) is the family of arc-locally in-semicomplete digraphs (arc-locally outsemicomplete digraphs). The intersection of the families of *H*1 free digraphs and the H_2 -free digraphs is the family of arc-locally semicomplete digraphs. The *H*₃-free digraphs are the 3-quasitransitive digraphs, and the *H*4-free digraphs are the 3-quasianti-transitive digraphs.

The absorption in digraphs is a dual concept to domination in graphs. Let *D* be a digraph, a set of vertices $S \subseteq V(D)$ is an *absorbent* set if for every vertex $w \in V(D) \setminus S$ there is an arc $wv \in A(D)$ with $v \in S$. A subset of vertices of a digraph *D* is a *kernel* if it is an independent set and an absorbing set. Von Neumann and Morgenstern introduced kernels in digraphs in 1944. In 1973, Chvátal proved that it is a \mathcal{NP} -complete problem to decide whether an arbitrary digraph has a kernel. In-

teresting surveys of kernels in digraphs can be found in [2, 4]. The *CKI-digraphs* is a nice family of digraphs. A digraphs *D* such that every proper induced subdigraph of *D* has a kernel is said to be *critical kernel imperfect digraph* (CKI-digraph) if the digraph *D* does not have a kernel and *kernel perfect digraph* (KPdigraph) if the digraph *D* has a kernel. If *D* is a CKI-digraph (or a KP-digraph), then *D* has no proper induced CKI-subdigraph. Galeana-Sánchez and Neumann-Lara proved in [3] that if *D* has no induced CKI-subdigraph, then *D* is KP, and hence, *D* has a kernel. Therefore, it is useful to characterize the CKI-digraphs.

In this talk, we present a characterization of the CKI-digraphs of some families of generalized tournaments. We use the connection between the KP digraphs and the perfect graphs to characterize the multipartite semicomplete CKI-digraphs and the quasitransitive CKI-digraphs. Moreover, we characterize the round CKI-digraphs and we use the characterization of the locally semicomplete digraphs by Bang-Jensen to characterize the locally semicomplete CKI-digraphs. We characterize an extend family of H_i -free digraphs defined y Bang-Jensen. We prove that, for $i =$ 1, 2, 3, we characterize the H_i -free CKI-digraph; and for $i = 4$ we characterize the H_4 -free CKI-digraphs satisfying that every H_4 path has at least two diagonals. It remains as an open problem to characterize the *H*4-free CKI-digraphs.

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Decomposition of cubic graphs

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This is a joint work with Dong Ye (Middle Tennessee State University).

It was conjectured by Hoffmann-Ostenhof that the edge set of every cubic graph can be decomposed into a spanning tree, a matching and a family of cycles. (Note that a matching and a set of cycles desired here are not necessarily spanning subgraphs.) We prove that the conjecture is true for 3-connected plane cubic graphs and 3-connected cubic graphs on the projective plane.

Finding clique pseudo-inverses of graphs

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Joint work with Pablo De Caria. Universidad Nacional de la Plata y CONICET.

The *clique graph K*(*G*) of a graph *G* is the intersection graphs of all its (maximal) cliques. Iterated clique graphs are then defined by $K^0(G) = G$ and $K^{n+1}(G) = K(K^n(G))$. A graph *G* is a *clique pseudo-inverse* of *H* if $K^3(G) \cong K^2(H)$. Here we present our results on clique pseudo-inverses and their relationship with separation problems on classes of iterated clique graphs.

Perspectives in distributed computing

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An introduction to the role of topology in distributed computing.

Toric ideals of graphs

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Joint work with Isidoro Gitler and César Guadarrama.

Let *G* = (*V*; *E*) be a graph, where $V = \{x_1, ..., x_n\}$ and $E =$ {*y*1, ..., *ym*}. We take *φ* the graded homomorphism of *k*-algebras

$$
\phi: k[y_1, ..., y_m] \rightarrow k[x_1, ..., x_n],
$$

induced by, $\phi(y_i) = x_k x_j$, where $y_i = \{x_k, x_j\}$. The kernel of ϕ , denoted by *PG*, is called the toric ideal associated to *G*. In this talk we study the minimal systems of generators of *PG*. Furthermore we characterize when P_G is a complete intersection. Finally we show a class of complete intersection obstructions.

Graph-like continua and infinite matroids

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In the last 20 years or so, there has been a lot of work done on extensions of results from finite graphs to compactifications of infinite graphs. These have found a natural home in the concept of a graph-like continuum: a compact, connected topological space *X* with a closed, totally disconnected subset *V* (the vertices) so that $X\setminus V$ consists of disjoint homeomorphs of the real line (the edges). The notion of a cycle space extends nicely to this context, as do spanning trees. The fundamental cycles relative to a spanning tree make a basis of the cycle space. The planarity characterizations of MacLane and Whitney also hold for graphlike continua. With the algebra behaving so well, it was natural to consider matroids. Higgs' notion of B-matroid turns out to be the right definition of an infinite matroid; many of the equivalent definitions of matroid generalize to definitions of B-matroid that are all equivalent. There are interesting theorems that generalize results for finite matroids. For example, a 3-connected binary matroid has the property that its peripheral cycles span its cycle

space and a matroid has no $U(2, 4)$ -minor if and only if it is representable using binary vectors – with certain infinite sums allowed. There is a conjectured version of the Matroid Intersection Theorem that generalizes the difficult Aharoni-Berger version of Menger's Theorem for infinite graphs.

Uniform hypergraphs containing no grids and no triangle

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This is joint work with Zoltán Füredi.

A hypergraph is called an $r \times r$ *grid* if it is isomorphic to a pattern of *r* horizontal and *r* vertical lines, i.e., a family of sets

 $\{A_1, \ldots, A_r, B_1, \ldots, B_r\}$

such that $A_i \cap A_j = B_i \cap B_j = \emptyset$ for $1 \leq i \leq j \leq r$ and $|A_i \cap B_j|$ B_j | = 1 for $1 \le i, j \le r$. Three sets C_1, C_2, C_3 form a *triangle* if they pairwise intersect in three distinct singletons, $|C_1 \cap C_2|$ = $|C_2 \cap C_3|$ = $|C_3 \cap C_1|$ = 1, $C_1 \cap C_2 \neq C_1 \cap C_3$. A hypergraph is *linear*, if $|E \cap F| \leq 1$ holds for every pair of edges.

In this paper we construct large linear *r*-hypergraphs which contain no grids. Moreover, a similar construction gives large linear *r*-hypergraphs which contain neither grids nor triangles. Our results are connected to the Brown-Erdős-Sós conjecture (Ruzsa-Szemerédi's (6,3) theorem is a special case) on sparse hypergraphs, and we slightly improve the Erdős-Frankl-Rödl bound. One of the tools in our proof is Behrend's construction on large sets of integers containing to three-term arithmetic progression.

Book embeddings and decompositions of permutations

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This is joint work with József Balogh.

We recall that a *book with k pages* is the topological space *B^k* that consists of a line (the *spine*) plus *k* half-planes (the *pages*), such that the boundary of each page is the spine. A *k*-page book embedding (or simply a *k*-page embedding) of a graph *G* is an embedding of *G* into *B^k* in which the vertices are on the spine, and each edge is contained in one page. It is known that a book on *m* edges can be embedded into a book with $O(\sqrt{m})$ pages, but it is not known whether or not this bound is tight for *d*-regular graphs (for any fixed *d*). We will talk about our recent progress on this problem.

Hamilton cycles in truncated triangulations of closed surfaces

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This is a joint work with Michal Kotrbčík and Roman Nedela.

A truncated triangulation is a 3-valent map on a closed surface *S* arising from a triangulation of *S* by truncating each vertex, that is, by expanding it into a contractible cycle. Several years ago, Glover and Marušič employed truncation of highly symmetrical triangulations of orientable surfaces as a tool for proving the existence of Hamilton cycles in large classes of cubic Cayley graphs. We generalise their method by replacing high symmetry with a somewhat surprising weaker condition – upper embeddability of the dual cubic graph. Our result enables us to construct Hamilton cycles in much wider classes of cubic graphs, including new classes of cubic Cayley graphs.

A graph colouring version of the Borsuk-Ulam theorem

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Part of this talk is based on joint work with Tomas Kaiser.

Youngs proved that every quadrangulation of the projective plane is either bipartite or (at least) 4-chromatic. Last year, Tomas Kaiser and I extended the definition of quadrangulation to arbitrary dimension, and showed that every quadrangulation of the projective n-space is either bipartite or at least (n+2)-chromatic. Our proof uses the topological methods initiated by Lovasz in his proof of Kneser's conjecture. At the heart of all such proofs lies the Borsuk-Ulam theorem.

I will show that, somewhat surprisingly, the Borsuk-Ulam theorem is an easy consequence of the lower bound on the chromatic number of projective quadrangulations. This seems to suggest that all topological lower bounds on the chromatic number ought to follow from this bound, and I will show that this is indeed the case for the class of 'strongly topologically t-chromatic" graphs, which includes Kneser graphs, Schrijver graphs, and generalised Mycielski graphs.

On panchromatic digraphs

Ricardo Strausz *Instituto de Matem´aticas, UNAM* dino@math.unam.mx

We introduce and study the concept of panchromaticity.

On increasing and decreasing subsequences of permutations

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His talk is based on joint work with J.L. Alvarez Rebollar, J. Cra-

vioto and T. Sakai.

A well known results by Erdös and Szekeres, asserts that any permutation $\Pi(n)$ of the integers $1, \ldots, n$, always has an increa s is a decreasing sequence with at least \sqrt{n} elements. In this talk, we review a well known proof of the previous result, and play a bit wit it. We then study a similar problems arising from point sets in the plane in general position, and labelled with the integers 1, ..., *n*. In this case, we are interested in the existence of long simple paths such that the labels of its elements are always increasing or decreasing.

Chromatic number and long cycles

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Erd˝os conjectured that every triangle-free *k*-chromatic graph contains cycles of *k* 2−*o*(1) lengths. In this talk, I give a short proof of a stronger statement, namely that every triangle-free *k*-chromatic graph contains cycles of $(\frac{1}{4} - o(1))k^2 \log k$ consecutive lengths.

Solving Rota's Conjecture

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Rota's Conjecture asserts that the matroids representable over a fixed finite field can be characterised by a finite set of forbidden minors. This conjecture has recently been resolved in joint work with Jim Geelen and Bert Gerards. In the talk I will give an overview of the conjecture and some of the ingredients of its proof. The talk will be introductory and will assume no knowledge of matroids.

Another step towards Yuzvinsky's conjecture

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This is a joint work with Isidoro Gitler and Enrique Reyes (Department of Mathematics, Cinvestav).

An $r \times s$ matrix with entries in $\{1, 2, \ldots, n\}$ is said to be *intercalate* if all entries on each row are different, all entries on each column are different, and each 2×2 submatrix has either two or four different entries. Let $f(r, s)$ be the minimum value of *n* for which there exists at least one intercalate matrix of $r \times s$. In 1981, Sergey Yuzvinsky (University of Oregon) conjectured that *f*(*r*,*s*) coincides with Pfister's function. Since then, several special cases of the conjecture have been settled, in particular for small values of r ($r \leq 5$) or for special values of *r* and *s*. In 2007, Kee Yuen Lam (University of British Columbia) proved that Yuzvinsky's conjecture is true for $r = s$. Furthermore, his proof implies that the conjecture is true asymptotically for $\frac{2}{3}$ of the pairs (r, s) . In this talk, we will show an extension of Lam's proof which implies that the conjecture is true asymptotically for $\frac{5}{6}$ of the pairs (r, s) . We also prove that Yuzvinsky's conjecture is true for $r \leq 8$.

Generalizations of universal fixer graphs

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Let $G = (V, E)$ be a graph with *n* vertices and G' a copy of G . For a bijective function $\pi : V(G) \to V(G')$, define the prism πG of *G* as follows: $V(\pi G) = V(G) \cup V(G')$ and $E(\pi G) = E(G) \cup E(G)$ $E(G') \cup M_{\pi}$, where $M_{\pi} = \{u\pi(u) \mid u \in V(G)\}$. Let $\gamma(G)$ be the domination number of *G*. If $\gamma(\pi G) = \gamma(G)$ for any bijective function π , then *G* is called a universal fixer. Mynhardt and Xu conjectured that the only universal fixers are the edgeless graphs *Kn*.

A non-isolated vertex $x \in V(G)$ is called C_3 -free if *x* belongs to

no triangle of *G*. In [2] was proved that any graph *G* with *C*3-free vertices is not a universal fixer graph. Recently, in [4] the author proves the above conjecture for any graph.

In this talk we generalize the concept of graphical universal fixer for two different types of non-classical domination and show that in both cases the equivalent conjecture is not true.

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POSTERS

A characterization of graphs with crossing number at least 2

Alan Arroyo *University of Waterloo* amarroyo@uwaterloo.ca

Joint work with Bruce Richter.

We present a "simple" graph-theoretical characterization of 4 connected graphs having crossing number at least 2: A 4-connected non-planar graph has crossing number at least 2 if and only if for every pair of disjoint edges there are vertex disjoint cycles containing these edges. We will discuss the ideas involved in the proof of this result, and mention some of its consequences in the study of 2-crossing-critical graphs (graphs that have crossing number at least 2, but any subgraph has crossing number 1 or 0).

Achromatic number of gramineous bipartite graphs

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Joint work with Laura Elena Chávez Lomelí and Rafael López Bracho.

The achromatic number of a graph is the maximum number of colors for which there is a proper vertex coloring such that every pair of colors appears in at least one edge, this kind of coloring is also known as complete coloring. There are few graph classes for which the achromatic number is known as the problem as been proved to be NP-hard even for trees. Approximation algorithms are known for some graph classes.

A gramineous graph is a birregular graph (the set of vertex degrees has at most two elements) in which a collection of disjoint paths can be found such that if the edges of the paths are contracted a complete graph is produced. The class of gramineous graphs has some interesting elements, in particular gramineous bipartite graphs have been discovered.

In this work we present a subclass of bipartite gramineous graphs for which the achromatic number has been exactly determined.

On Cohen-Macaulay and shellable graphs with the König property

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Joint work with Enrique Reyes.

Let $G = (V; E)$ a simple graph where $V(G) = \{x_1, ..., x_n\}$ is the vertex set and $E(G)$ is the edge set. The edge ideal of G is the monomial ideal $I = (x_i x_j | \{x_i, x_j\} \in E(G))$ in the polynomial ring $R = k[x_1, ..., x_n]$ on a field *k*. The ring R/I is Cohen-Macaulay if its krull dimension equals its depths as a *R*-module. The simplicial complex associated to the ideal *I* is

$$
\Delta_I = \{ \{x_{i_1}, ..., x_{i_k}\} \mid i_1 < \cdots < i_k, x_{i_1} \cdots x_{i_k} \notin I \}.
$$

By the primary decomposition of the ideal *I* and the Stanley-Reisner correspondence the elements of *∆^I* are the stable set of *G*. It is well-known, if *G* (or *∆^I*) is pure shellable then *R*/*I* is Cohen-Macaulay ring. A graph has the könig property if its matching number equals its transversal number or the height of *I*. We will show a combinatorial characterization of the Cohen-Macaulay graphs when they are könig. Furthermore, we will prove that if *G* is könig, then the properties Cohen-Macaulay, pure shellable and unmixed vertex decomposable are equivalent.

Algorithms for the Subset Query Problem

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Let *C* be a collection of *n* subsets of *U*, with $|U| = d$. The subset query problem consists in determining, for a given $Q \subseteq U$, if a subset of *Q* exists in *C*. This problem can be solved in *O*(*nd*/*m*) time and $O(nd^m)$ memory for any $m \leq n$, or in $O(n/2^m)$ time and $n2^{O(dlog(d)^2 \sqrt{(m/log(n))})}$ memory for any *m*. We explore new algorithms that restrict their memory consumption to *O*(*nd*) and $O(nd^2)$.

Some results on 4**-transitive digraphs**

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Let *D* be a digraph with set of vertices *V*(*D*) and set of arcs *A*(*D*). We say that *D* is *k*-transitive if for every pair of vertices $u, v \in V(D)$, the existence of a *uv*-path of length *k* in *D* implies that $(u, v) \in A(D)$. If $k = 2$ then we simply say that the digraph is transitive. Transitive and 3-transitive digraphs have been proved to be well behaved families, in the past some conjectures have been verified and some \mathcal{NP} -complete problems have been proved to be polynomial time solvable for these families.

A *k*-kernel of a digraph *D* is a *k*-independent and (*k* − 1)-absorbent vertex subset of *D*. In this work the 4-transitive digraphs that have a 3-kernel and a 2-kernel are characterized (recall that in general it is \mathcal{NP} -complete to determine wether a digraph has a *k*-kernel for every $k \geq 2$). Using the latter characterization, a proof of the Laborde-Payan-Xong conjecture for 4-transitive digraphs is given. This conjecture establishes that for every digraph *D*, an independent set can be found such that it intersects every longest path in *D*. Also, it is simple to verify Seymour's Second Neighborhood Conjecture and the Caccetta-Häggkvist Conjecture for 4-transitive digraphs.

Interconnetion Networks

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Interconnetion Networks.

Circulant digraphs with kernels

Guadalupe Gaytán Gómez *UAM-Iztapalapa* gaytan101295@gmail.com

Let *D* be a digraph, $V(D)$ and $A(D)$ will denote the set of vertices and arcs of *D*, respectively. A set $N \subseteq V(D)$ is said to be a *kernel* of *D* if it satisfies the following two conditions: (i) for every pair of different vertices $u, v \in N$ there is no arc in *D* between them and; (ii) for every vertex $x \in V(D) - N$ there is a vertex $y \in N$ such that there is an *xy*-arc in *D*.

For an integer $n \geq 2$ and a set $S \subseteq \{1, 2, ..., n-1\}$, the *circulant digraph* $C_n(S)$ is defined as follows: $V(C_n(S)) = \{1, 2, ..., n\}$ and $A(C_n(S)) = \{(i, i + j \pmod{n}) : 1 \le i \le n, j \in S\}.$

Duchet conjectured that every digraph *D*, which is not an odd cycle and which does not have a kernel, contains an arc *e* such that *D* − *e* has no kernel either. Apartsin, Ferapontova and Gurvich found a counterexample to this conjecture. They proved that the circulant digraph $C_{43}(1, 7, 8)$ has no kernel, but after deletion of any arc in this digraph a kernel will appear. *C*43(1, 7, 8) is the only known counterexample to the Duchet conjecture; Gurvich suspects that there is an infinite family of such circulant digraphs. Gurvich, et al. also proved that *Cn*(1, 7, 8) has a kernel if and only if $n \equiv 0 \pmod{3}$ or $n \equiv 0 \pmod{29}$.

In this work it is proved that $C_n(1, 2, \ldots, k)$, $n \ge 2k + 1$ has a kernel if and only if $n \equiv 0$ *mod*($k + 1$).

On the k-kernel problem and its complexity in some classes of digraphs

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Let *D* be a digraph with vertex set *V* and arc set *A*, and let $k \geq 3$ be an integer. We say that *D* is cyclically *k*-partite if there exists a partition of *V* in *k* subsets V_1, V_2, \ldots, V_k such that, if $vu \in A$, then $v \in V_i$ and $u \in V_{i+1}$, where the subscripts are taken modulo *k*. A subset *X* ⊂ *V* is *k*-independent if $d(v, u) \geq k$ for every pair of vertices *v*, *u* in *X*. A subset $Y \subset V$ is *l*-absorbent if for each $v \in V$, there is a vertex $u \in Y$ such that $d(v, u) \leq l$. A subset $K \subseteq V$ is a *k*-kernel if it is *k* independent and (*k* − 1)-absorbent.

Hell and Hernández-Cruz proved that the problem of finding a 3kernel of a digraph is NP-complete even if restricted to the family of cylically 3-partite digraphs with circumference 6. In this talk I will show that for every integer *k* greater than 3, the problem of finding a *k*-kernel for a digraph is NP-complete even if restricted to the family of cyclically *k*-partite digraphs with circumference 2*k*.

Also, a result from the same work states that if a digraph admits a cyclical 3-partition (V_1, V_2, V_3) such that there is at least one partite set without sinks, then *D* has a 3-kernel. This result is generalized for cyclically *k*-partite digraphs. Additional sufficient conditions for the existence of *k*-kernels in cyclically *k*-partite digraphs are discussed.

The acyclic disconnection spectra of grids

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Let $D = (V, A)$ be a digraph with vertex set V and arc set A. In 1999, Neumann-Lara defined the acyclic disconnection of *D*, $\vec{\omega}(D)$, to be the maximum number of colors in a proper coloring of *V* not producing a properly colored directed cycle. Since then, only a fistful of papers have been devoted to this subject, most of them analyzing this parameter on tournaments and bipartite tournaments.

Given a graph *G*, we define the *acyclic disconnection spectrum* of *G*, $S_{\overrightarrow{\omega}}(G)$, to be the set

$$
S_{\vec{\omega}}(G) = \{ \vec{\omega}(D) \colon D \text{ is an orientation of } G \}.
$$

Since the acyclic disconnection of a digraph is the sum of the acyclic disconnections of its strong components, it results natural to define the *strong acyclic disconnection spectrum* of *G, S* $_{S\overrightarrow{\omega}}(G)$ *,* to be the set

 $S_{\vec{S}\vec{\omega}}(G) = {\vec{\omega}(D) : D \text{ is a strong orientation of } G}.$

Intuitively, there is a relation on the cyclic structure of a graph *G* and its acyclic disconnection spectrum. Nonetheless, very little is known about the acyclic disconnection spectrum in general digraphs, and so, this relation remains only at the intuition level.

This talk is divided in two parts. In the first part we will prove that the problem of determining the acyclic disconnection of a digraph is \mathcal{NP} -complete, even for bipartite digraphs. In the second part we will calculate the exact strong acyclic disconnection spectrum for every grid graph $G = P_n \square P_m$, $n, m \geq 2$. Also, depending on time constraints, we will discuss the strong acyclic disconnection spectra of complete graphs.

Dichromatic number of infinite families of circulant tournaments

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The *dichromatic number* of a digraph *D* is defined to be the minimum number of colors required to color the vertices of *D* in such a way that every chromatic class induces an acyclic subdigraph in *D*. A tournament is an orientation of the complete graph. Let *m* be a positive integer. A *circulant tournament* $\mathcal{L}_{2m+1}(J)$ is defined by $V(\overrightarrow{C}_{2m+1}(J)) = \mathbb{Z}_{2m+1}$ and

$$
A(\overrightarrow{C}_{2m+1}(J)) = \{(i,j) : i,j \in \mathbb{Z}_{2m+1} \setminus \{0\} \text{ and } j-i \in J\},\
$$

where *J* ⊆ $\mathbb{Z}_{2m+1} \setminus \{0\}$ and $|J \cap \{a, -a\}| = 1$ for every *a* ∈ $\mathbb{Z}_{2m+1}\setminus\{0\}$. The set *J* is called the *symbol set* (the elements of *J* are sometime called the *jumps*) of $\overline{C}_{2m+1}(J)$. The *cyclic* circulant tournament is denoted by $\overline{C}_{2m+1}(1, 2, ..., m)$. Denote by $\overline{C}_{2m+1} \langle k \rangle$ the circulant tournament obtained from the cyclic tournament by reversing one of its jumps, that is,

$$
\overrightarrow{C}_{2m+1} \langle k \rangle = \overrightarrow{C}_{2m+1} (1, 2, \ldots, -k, \ldots, m)
$$

for some $k \in \{1, 2, ..., m\}$. With this definition, the cyclic tour- $\overrightarrow{C}_{2m+1}(1,2,...,m)$ becomes $\overrightarrow{C}_{2m+1}\left\langle \varnothing\right\rangle$. This poster shows the dichromatic number of $\overline{C}_{2m+1} \langle k \rangle$ for every $k \in \{1, 2, ..., m\}.$

Reproducing the friendship graph

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In this study, we analogize the Facebook graph for individual users in order to reconstruct the friendship structure for each one of the users. We compute numerous features of the Facebook graph including the path lengths, clustering and mixing patterns, as well as several other indicators to establish an apriori relation of "friendship". These indicators are age range, education, hometown and likes. This a-priori friendship was compared with actual data in order to customize and evaluate our algorithms.

With this aim, several algorithms were programmed using python language and the networkx library. The results suggest that our algorithms can predict with a high degree of certainty when two people are friends or not from the variables studied. These results are linked to the user's social exposure, for example our results improve considerably when the Facebook users have 20 or more likes.

A perspective on protein structures: Prediction of knots for the protein folding problem

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Topology and, in particular, knot and link theory of closed space curves have been used extensively to elucidate the intertwining of biomolecules. In particular, recent developments in polynomial invariants for links and knots have been used to describe the structures of proteins. Evidence for knot-promoting loops in proteins suggests that it should be possible to predict the tendency of a given protein to form knots. This work presents a proposal for the prediction of knots given an amino acid sequence. The knots thus found would be of help in early steps of algorithms for protein structure prediction and for advancing our understanding of protein functionality.

A memetic algorithm for the multi-index assignment problem

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Joint work with Carlos Valencia, Francisco Zaragoza, Antonin Ponsich and Roman Mora.

An assignment problem deals with the question of how to assign a set X of size n to another set Y of the same size. The multi-index assignment problem arises when one requires an assignment between elements of three or more disjoint sets. For M disjoint sets the problem is called the axial M-index assignment problem (axial MAP). This problem is known to be NP-hard for three or more sets (Karp, 1972). A memetic algorithm is a combination of a genetic algorithm with some local search heuristics. We pesent some experimental results as an approach to improve state of the art heuristics for this problem.

Applying the circular chromatic index to cyclic production systems

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Joint work with José de Jesús Rodríguez Martínez.

The circular chromatic number $\mathcal{X}_c(G)$ of a graph *G* is a refinement of the chromatic number of a graph. That parameter was introduced by Vince in 1988. The circular chromatic index $\mathcal{X}'_c(G)$ of a graph with weights on its edges is applied for task scheduling in systems whose production process has a cyclic character.

The concept of graph circular coloring is applied to develop a model for the special case of an open shop scheduling problem. In this problem, there are some independent jobs to be processed in a shop with dedicated renewable resources. Each job consists of several tasks with no precedence restriction. Each task is processed without preemption. The processing time of the tasks is known. Some tasks can be shared by more than one job and the process may be repeated more than once.

This problem can be modeled beginning with a bipartite weighted graph, whose vertices of one partition represent jobs and of the second partition represent processors and resources. An edge corresponds to the task associated with the job *i* and the processors or resources *j*, the weight on the edge *ij* is the time necessary for the task *ij*. The objective is to developed a schedule which yields the minimal makespan length of all jobs, as well as the number of cycles.

Extremal graphs free of quadrilaterals

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In 1958, Rieman proved that if a graph has *n* vertices, *e* edges and no 4-cycles, then $e \le n(1 + \sqrt{4n-3})/4$, but this is known not to be sharp. Erdös, Rényi and Sós later showed that this asymptotically correct.

We give an improved bound.

On algebraic independence of clutters

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Joint work with Enrique Reyes.

We say that a clutter is algebraically independent if its set of edges from an algebraically independent set in the polynomial ring, in this work we study clutters which satisfy this property and we present some relations between algebraic independence and asymptotic properties like the strong persistence property and the persistence property.

On the achromatic and pseudoachromatic number for some designs

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Joint work with G. Araujo, Gy. Kiss and C. Rubio-Montiel.

A *design* $\mathcal{D} = (\mathcal{V}, \mathcal{B}, I)$ with parameters $v > 1$, b, κ and r (positive integers), is a system D consisting of a set V of v points, a set B of *b* blocks, and an incidence relation *I*, a subset of $V \times B$. We say that *x* is incident to *y* (or *y* is incident to *x*), if and only if, the ordered pair (x, y) is in *I*. Furthermore D must satisfy the following axioms:

- 1. Each block of D is incident to exactly *κ* distinct points of D.
- 2. Each point of D is incident to exactly *r* distinct blocks of D.
- 3. If *x* and *y* are distinct points of D, there is exactly one block of D incident to both *x* and *y*.

Given a design $\mathcal{D} = (\mathcal{V}, \mathcal{B})$, a *block-coloring* (for short *coloring*) of D with *k* colors is a surjective function $\zeta : \mathcal{B} \to \{1, \ldots, k\}.$ A coloring of D with *k* colors is *proper*, if for any two different blocks $B, B' \in \mathcal{B}$ with $B \cap B' \neq \emptyset$ satisfies $\zeta(B) \neq \zeta(B')$. The *chromatic index* $\chi'(\mathcal{D})$ of $\mathcal D$ is the smallest *k* such that there exists a proper coloring with *k* colors of D.

A coloring of a design D is *complete*, if each pair of colors appears on at least a point of D. The *achromatic index α* ′ (D) of D is the largest number *k* for which there exists a complete and proper coloring of D with *k* colors. The *pseudoachromatic index ψ* ′ (D) of D is the largest number *k* for which there exists a complete coloring of D with *k* colors. In the present poster we present some results about the achromatic and pseudoachromatic indices for some types of designs, giving bounds and exact values.

CIO **deficiency of graphs**

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Joint work with I. Gitler and E. Reyes.

Let *G* be a finite simple graph. We consider for each oriented graph $D = G_{\mathcal{O}}$ associated to an edge orientation $\mathcal O$ of *G* the toric ideal P_D . The graphs with the property that P_D is a binomial complete intersection for each edge orientation $\mathcal O$ of G are called CI $\mathcal O$ graphs. We introduce the CIO deficiency $\epsilon(G)$. This invariant has the property that graphs with CIO deficiency zero are exactly CIO graphs. We prove that the CIO deficiency is increasing under induced subgraphs on connected graphs. We show that the CIO deficiency is additive with 0, 1-clique-sums. We define *k*-chorded-theta graphs and examine their CIO deficiency.

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