Anti-Ramsey Colorings in Several Rounds

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(joint work with Aart Blokhuis, András Gyárfás and Miklós Ruszinkó)

For positive integers $k \leq n$ and t let $\chi^t(k,n)$ denote the minimum number of colors such that at least in one of the total t colorings of edges of K_n all $\binom{k}{2}$ edges of every $K_k \subseteq K_n$ get different colors. Generalizing a result of Körner and Simonyi, it is shown in this paper that $\chi^t(3,n) = \Theta(n^{1/t})$. Also two-round colorings in cases $k > 3$ are investigated. Tight bounds for $\chi^2(k,n)$ for all values of k except for $k = 5$ are obtained. Conversely, let $t(k,n)$ denote the minimum number of colorings such that – having the same $\binom{k}{2}$ colors in each coloring – at least in one of the total t colorings of K_n all $\binom{k}{2}$ edges of every $K_k \subseteq K_n$ get different colors. It is also shown, that for $k = n/2$ $t(k,n)$ is exponentially large. Several related questions are investigated.