On the Heterochromatic Number of Stars

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Let K_n be the complete graph of order n and $K_{1,q}$ the star of q edges, with $n > q \geq 2$. Now let c be the minimum number such that every surjective c–coloring of the edges of K_n procudes at least one copy of $K_{1,q}$ all whose edges have different colours (i.e., an heterochromatic copy of $K_{1,q}$).

In 1996, Y. Manoussakis, M. Spyratos, Zs. Tuza and M. Voigt conjectured that

$$c = \left[\frac{n(q-2)+4}{2} \right].$$

We disprove this conjecture by proving that

$g(nq) \mid 1 \geq c \geq g(n,q)$ if n and q are odd and $\frac{r(n,n-q+2}{n-q+2} \geq \frac{1}{2}$;

and

$c = g(n,q)$ in the remaining case, where

$$g(n,q) = \left[\frac{n(q-2)}{2} \right] + \left[\frac{n}{n-q+2} \right] + 1$$

and $r(n,n-q+2)$ is the residue of n mod $n-q+2$.