Some Combinatorial Problems of Integral Quadratic Forms

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Let $q : \mathbb{Z}^n \to \mathbb{Z}$ be an integral quadratic form of the shape

$$q(x) = \sum_{i=1}^{n} x(i)^2 + \sum_{i<j} q_{ij} x(i)x(j)$$

Called a *unit form*. Such forms play an important role in many fields of mathematics, in particular, as forms associated to algebraic structures as Lie algebras, finite dimensional algebras, etc.

In our work we describe algorithms to decide whether or not a given unit form is *weakly* non–negative (i.e. $q(x) \geq 0$ for all $n\mathbb{N}^n$) or non–negative. In case q is non–negative, we show that there is an invertible transformation $T : \mathbb{Z}^n \to \mathbb{Z}^n$ such that $qT(x) = \bar{q}(x_1, ..., x_{n-s})$ where $\bar{q}$ is a positive unit form associated to a *Dynkin* diagram.