University Admissions *via* Graphs

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An *admissions problem* or *admissions game* (Γ, q) is specified by a directed graph Γ defined over a grid, and positive integers q, as follows. There are two distinct, finite sets of players, $U = \{u_1, u_2, \ldots, u_{|U|}\}$ (the "universities") and $A = \{a_1, a_2, \ldots, a_{|A|}\}$ (the "applicants"), each player has a strict preference order over those players of the opposite set whom it, she or he considers to be acceptable, and each university u has a quota q_u of applicants it may accept.

The solutions of interest are the *stable assignments*: those feasible assignments having the property that no pair of agents $u \in U, a \in A$ not matched can both improve their situations by being matched.

A graphical algorithm finds a reduced graph, containing all stable assignments, in which the *applicant–optimal* μ_A and *university-optimal* μ_U stable assignments are immediate.

The fine structure of stable assignments shows that, contrary to current belief, the admissions problem is virtually equivalent to the marriage problem: there *is* a symmetry in the qualitative properties of the optimal stable assignments μ_A and μ_U. Each is characterized as the unique mechanism that is either "monotone," "strategy-proof" or "Pareto optimal" on one side of the market.

A new graphical characterization of stability leads to a description of the *stable admissions polytope*, the convex hull of the stable assignments expressed in terms of linear inequalities.