About the Dimension of Digital Quotients of $\mathbb{R}^n$

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Alexandroff T_0–spaces, whose category is equivalent to that of posets, have been studied recently as topological models of the supports of digital images and as discrete models of continuous spaces in theoretical physics. A problem on which research has been focused recently, is that of the dimension of a digital space. In previous works we defined a topological dimension function for Alexandroff spaces called the *Alexandroff dimension*, which is essentially the small inductive dimension *ind* from classical general topology. We proved that for an Alexandroff T_0 space, this dimension coincides with the height of the corresponding poset.

In this talk we apply the Alexandroff dimension to a digital space which is obtained as a quotient of the Euclidean space $\mathbb{R}^n$, following a construction proposed by E. Kronheimer: Starting with a locally finite collection W of pairwise disjoint regular open subsets of $\mathbb{R}^n$ whose union is dense in $\mathbb{R}^n$ (this family is called *fenestration* of $\mathbb{R}^n$, we construct a quotient $X(W)$ of $\mathbb{R}^n$ called *digital space* by extending W to a partition $X(W)$ of $\mathbb{R}^n$. The $X(W)$ is topologized by the natural quotient topology; and the natural projection from $\mathbb{R}^n$ to $X(W)$ is assumed to be open. Furthermore, $X(W)$ turns out to be unique up to homeomorphism if a certain minimality condition is also imposed. Kronheimer has shown that for an arbitrary fenestration W of $\mathbb{R}^n$, the digital space $X(W)$ is a locally finite (and hence Alexandroff) T_0 space. We show examples of fenestrations of $\mathbb{R}^n$ (whose elements are clearly n–dimensional subsets of $\mathbb{R}^n$), which give rise to digital spaces of Alexandroff dimensions different from n. However, we prove that, if W is a fenestration, each of whose elements is a bounded convex subset of $\mathbb{R}^n$, then the Alexandroff dimension of the digital space $X(W)$ is equal to n. To prove this, the digital space $X(W)$ is explicitely constructed. The local finiteness of W and the convexity of its elements are essential for our proof.