Iterated Clique Graphs and Posets

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The *cliques* of a graph G are its maximal complete subgraphs. The *clique graph* kG has the cliques of G as vertices and two different cliques of G are joined by an edge in kG whenever they share at least one vertex. The *iterated cliques graphs* of G are defined by $K^{n+1}G = kk^nG$. A graph G is said to be *clique divergent* if the order of k^nG tends to infinity with n, and *clique null* if some k^nG is trivial.

For the first–known examples of clique divergent graphs, the divergent sequence $o(k^nG)$ grows superexponentially. Recently, we have found examples of clique divergent graphs whose growth is linear.

On the other hand, S. Hazan and V. Neumann–Lara proved the following result: If P is a finite poset and the comparability graph G of P is clique null, then P has the *fixed point property:* every endomorphism of P has a fixed point. They also asked the question whether the converse is true.

We will solve this last question in the negative by exhibiting (from B. Schrder's list of all the posets with eleven elements that have the fixed point property and no retractable point) a finite poset P such that the comparability graph G of P is clique divergent and $o(k^nG)$ is a linear function of n. Indeed, the ninth iterated clique graph k^9G is a member of an interesting family of graphs f which is closed under the clique graph operator k. Two ingredients enter into the construction of the graphs of F, and one of them is a proper circular–arc graph satisfying an old condition introduced by A. Tucker (no three arcs cover the entire circle). The fact that k^9G is in F is established with the aid of a computer, and from there the theory gains control: at least under certain conditions, it is possible to decide whether a graph in F is clique divergent.