New trends in combinatorial geometry

Janos Pach
New York University

We survey several recent developments in combinatorial geometry. The presented results are divided into four categories. (A) Quantitative Helly–Erdős–Szekeres–Tverberg–type theorems, (B) Distribution of distances among n points, (C) Bounds on the number of incidences between points and curves or surfaces, (D) Crossing numbers of graphs. As an example of a problem belonging to (A), we present a detailed proof of the following result. Let $P_1, \ldots, P_{d+1}$ be pairwise disjoint n-element point sets in general position in d-space. Then there exist a point O and suitable subsets $Q_i \subseteq P_i$ ($i = 1, \ldots, d+1$) such that $|Q_i| \geq c_d |P_i|$ and every d-dimensional simplex with exactly one vertex from each Q_i contains O in its interior. Here c_d is a positive constant depending only on d. This generalizes a theorem of Vrećica and Živaljevič. The proof is based on the Szemerédi Regularity Lemma.