

Some topics in matroid representation theory

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A matroid is regular if it can be represented over the rationals by a matrix A with the property that all subdeterminants of A are in $\{0, \pm 1\}$. It is a classical result of Tutte that if \mathcal{F} is a set of fields containing $GF(2)$, then the class of matroids representable over all fields in \mathcal{F} is either the class of regular or binary matroids. What if \mathcal{F} is a set of fields containing $GF(3)$? It turns out that essentially three new classes arise. And these can all be characterised as classes of matroids representable over certain fields by certain types of matrices. Examples of consequences of these characterisations are: a matroid is representable over $GF(3)$ and the complex numbers if and only if it is representable over $GF(3)$ and $GF(7)$; a matroid is representable over $GF(3)$ and the rationals if and only if it is representable over $GF(3)$ and $GF(5)$, and this holds if and only if it is representable over all fields of odd characteristic; a matroid is representable over all fields except possibly $GF(2)$ if and only if it is representable over $GF(3)$ and $GF(8)$.

The talk will outline these results, make connections with cross ratios and consider directions for future research.