Generating upper bounds for the sums of squares formulae problem

Adolfo Sánchez
IIMAS–UNAM

The number $r *_Z s$ is the smallest integer t such that there exists a formula of the form $(x_1^2 + \cdots + x_r^2)(y_1^2 + \cdots + y_s^2) = z_1^2 + \cdots + z_t^2$, where each z_i is a bilinear form in the sets of indeterminates X and Y with integer coefficients. Finding a formula of this type is equivalent to obtaining an $r \times s$ *consistently signed intercalate matrix* of type $[r, s, t]$. Most of the best upper bounds known for $r *_Z s \, (1 \leq r,\, s \leq 64)$ are obtained by juxtaposing two of these matrices of smaller size. However, an "irreducible" juxtaposition of 5 matrices was recently given, so as to provide the best upper bounds known for some $r *_Z s$. Here we describe all the irreducible configurations that juxtapose k matrices, for $k \leq 9$. Also, we show an algorithm that, for given $r, s \geq 1$, and a set C of upper bounds on $i *_Z j \, (i \leq r,\, j \leq s$, and $i + j < r + s)$, produces the juxtaposition of $k \leq 7$ matrices that yields the lowest bound on $r *_Z s$ induced by C.