Flows, colourings and the lonely runner conjecture

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"Suppose that n runners run many laps around a circular track of unit length. The runners have constant, distinct speeds. Is it true that for each runner r, there is a time at which r is at distance at least $1/n$ from all the other runners?"

This "Lonely Runner Conjecture" (LRC), posed by Wills in 1968, is still open for $n > 5$. This conjecture is related to "Diophantine Approximation", "Geometric Visibility", "Finite Difference Sets" and "Colourings and Flows in Graphs and Matroids".

We outline a new short proof of the case $n = 5$ of LRC, and use this to derive results regarding flows in graphs and matroids, including:

"If G has a nowhere-zero flow having at most $k - 1$ distinct values, then G has a nowhere-zero k-flow."