Structure of Topologically Closed Classes of Trees

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This is joint work with Yared Nigussie and includes work done earlier with Seymour and Thomas. A theorem of Joseph Kruskal from 1960 states that finite trees under topological containment are well-quasi-ordered; i.e., that in any infinite set of finite trees, some tree is topologically contained in another. In a well-quasi-order (Q,¡=), coherence, the property that any two objects in Q are contained in a third, is associated closely with structure. The fact that there are only finitely many distinct maximal coherent inclusion-closed subsets of (Q,¡=) reflects the intuition that the number of structure types of members of Q should be finite. To test this hypothesis, a constructive process based on an encoding of topologically closed families of trees by sets of "bits" is given, subject to axioms that ensure uniqueness of the encoding. The tree families can be inductively constructed from the "bit" description. As it turns out the "bit" description leads to a finite labelled tree which itself determines the tree family. It is hoped that this result is typical of the general situation for minor-closed classes L of finite graphs. If true his would confirm certain conjectures about the structure of the graphs in L related to finite graphs being well-quasi-ordered.