Half integrality of postman polyhedrons[1]

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Given a strongly connected mixed graph G (with vertex set V, edge set E, arc set A) and nonnegative integral vectors $c, l, u \in \mathbb{Z}_+^{E \cup A}$, the *Bounded Postman Problem* (BPP) consists of finding a closed circuit C in G that traverses each $e \in E \cup A$ at least l_e and at most u_e times, with minimal cost $c(C)$.

The BPP is a generalization of several other problems, including the *Eulerian Walk Problem* where $l_e = u_e = 1$ for all $e \in E \cup A$, the *Undirected Postman Problem* (UPP) where $A = \emptyset$, the *Directed Postman Problem* (DPP) where $E = \emptyset$, the *Mixed Postman Problem* (MPP) where $l_e = 1, u_e = +\infty$ for all $e \in E \cup A$, the *Rural Postman Problem* (RPP) where $l_e \in \{0, 1\}, u_e = +\infty$ for all $e \in E \cup A$, and the *Restricted Postman Problem* where $l_e = 1, u_e \in \{1, +\infty\}$ for all $e \in E \cup A$. The *Windy Postman Problem* (WPP) is a related problem where G is undirected, and the cost of traversing an edge depends on the direction in which it is traversed.

Each of these problems can be formulated as an integer program. Some of these integer programs have linear relaxations that define integral polyhedrons (UPP, DPP). However, it is also known that some others define half-integral polyhedrons (MPP, RPP, WPP). The aim of this talk is to provide a new and simple proof of the half-integrality of the BPP linear relaxation. From this proof, many other related results about the linear relaxations of postman problems can be easily derived.

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