The Theta Body and Perfect Graphs

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The minimal imperfect graphs have the following structure. There exist integers p, q such that G has $pq + 1$ nodes and for each node $v \in V(G)$, $G - v$ can be covered both by q stable sets (independent sets) and by p cliques (complete graphs). A graph is *partitionable* if it has this property for some integers p, q. Clearly the recognition problem for such graphs is in NP as we need only present the appropriate cliques and stable sets as a certificate. We discuss characterizations of partitionable graphs which involve linear and semi-definite programming. One of these results in a polytime recognition algorithm based on the Ellipsoid method.

In order to recognize a perfect graph, one must ostensibly solve a maximum clique problem — generally hard but easy for perfect graphs — for every subset $S \subseteq V$. We also describe several characterizations of perfect graphs which lead to recognition algorithms based on solving a polynomially bounded number of problems which are hard in general but may be easy for perfect graphs. One of these subproblems is $\max\{\|x\|^2 : xQ(G)\}$ where $Q(G)$ is a convex body for which we have a polytime separation algorithm.