Approximating small vertex-connectivity problems via Set-Covers

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(joint work with Z. Nutov)

Given an undirected graph with costs on the edges, and an integer k, we consider the problem of finding a k–node connected spanning subgraph of minimum cost. For the general instance of the problem, the previously best known algorithm has approximation ratio $2k$. For $k \leq 5$, algorithms with approximation ratio lceil $(k+1)/2$ rceil are known. For metric costs Khuller and Raghavachari gave a $(2+2(k-1)/n)$–approximation algorithm. We obtain the following results.

1) An $I(k-k_0)$–approximation algorithm for the problem of making a k_0–connected graph k–connected by adding a minimum cost edge set, where $I(k) < k$ for $k \geq 7$.

2) A $(2+(k-1)/n)$–approximation algorithm for metric costs.

3) A lceil $(k+1)/2$ rceil–approximation algorithm for $k = 6, 7$.

4) A fast lceil $(k+1)/2$ rceil–approximation algorithm for $k = 4$.

The multiroot problem generalizes the min-cost k–connected subgraph problem. In the multiroot problem, requirements k_u for every node u are given, and the aim is to find a minimum–cost subgraph that contains max k_u, k_v internally disjoint paths between every pair of nodes u, v. For the general instance of the problem, the best known algorithm has approximation ratio $2k$, where $k = \max k_u$. For metric costs there is a 3–approximation algorithm. We consider the case of metric costs, and, using our techniques, improve for $k \leq 7$ the approximation guarantee from 3 to less than 2.5.