Clique Behaviour of Locally C_t Graphs.

Francisco Larrión
IMATE–UNAM

(joint work with V. Neumann–Lara and M.A. Pizaña)

We study the dynamical behaviour of graphs under the iterated application of the clique graph operator k, which transforms each graph G into the intersection graph kG of its (maximal) cliques. A graph G is k*-divergent* if the sequence of the orders of its iterated clique graphs $o(k^nG)$ tends to infinity with n. If this is not the case, then G is k*-convergent*: $k^nG \cong k^mG$ for some $m > n$.

A graph G is said to be a *locally C_t graph* if the neghbours of any vertex of G induce a cycle of length t (here, $t \geq 3$ is fixed). If $3 \leq t \leq 5$, there exists precisely one connected locally C_t graph, namely the tetrahedron, octahedron and icosahedron respectively. The terahedron is clearly k-convergent, while the other two are k-divergent (Neumann-Lara 1973, Pizaña 1999). For each $t \geq 6$ there is an infinite number of connected locally C_t graphs, and it was previously shown that every locally C_6 graph is k-divergent (Larrión and Neumann-Lara 1999, but part of this was reported in ACCOTA96).

In this work we show that, for each $t \geq 7$, every locally C_t graph G is k-convergent and indeed $kG \cong k^3G$. Most of our proof works in the more general setting of graphs with local girth at least 7: As a consequence we obtain also the k-convergence of the underlying graph G of any triangulation of a compact surface (with or without border) provided that any triangle of G is a face of the triangulation and that the minimum degree of the interior vertices of G is at least 7.