A Combinatorial Problem in Cyclic Abelian Groups

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(joint work with Oriol Serra and Balázs Szegedy)

In this talk we study the following two problems. First, let $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ be subsets of a finite Abelian group G. Is there any permutation $\pi \in S_k$ such that the sums $a_i + b_{\pi(i)}$ $(1 \le i \le k)$ are all distinct? In the second problem, A need not be a subset of G: repeated elements are allowed in the sequence $a_1, a_2, \ldots, a_k$.

Noga Alon gave a positive answer to the first problem in the case when $G = Z_p$ is a cyclic group of prime order, and also to the second problem, under the (necessary) assumption of $k < p$. With a novel application of the polynomial method in various finite and infinite fields we extend Alon's result to arbitrary cyclic groups of odd order in the first case (note that the result is not true for groups of even order), and for the groups $G = (Z_p)^\alpha$ and $G = Z_{p^\alpha}$ in the second case.